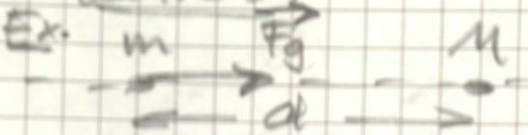
[Engineering Physics II: Electromagnetism] 8.28.29

Lecture 01:

Ex.

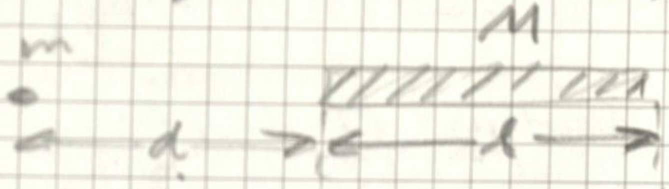


$$F_g = \frac{GMm}{d^2}$$

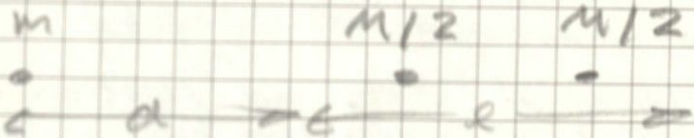
Direction of force on m coming from M?

$\longrightarrow$

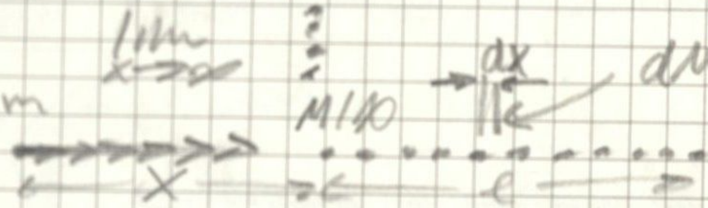
Ex. Introducing Calculus to Physics II



$$F_g = \frac{GmM}{d^2}$$



$$F_g = \sum_{0}^{i} \frac{GmM_i}{d_i^2}$$



$$dF_g = \frac{Gm\,dM}{x^2}$$

assume uniform dist

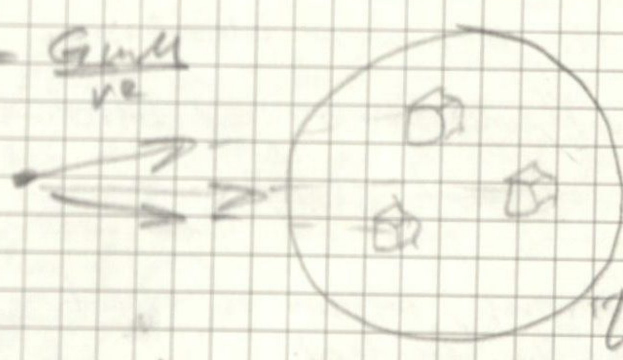
$$\frac{dM}{M} = \frac{dx}{l}$$

$$\Rightarrow dM = \left(\frac{M}{l}\right)dx$$

Units: $\left[\frac{M}{l}\right] = \frac{kg}{m}$

"Linear" density λ

Ex. $F_g = \frac{GmM}{r^2}$



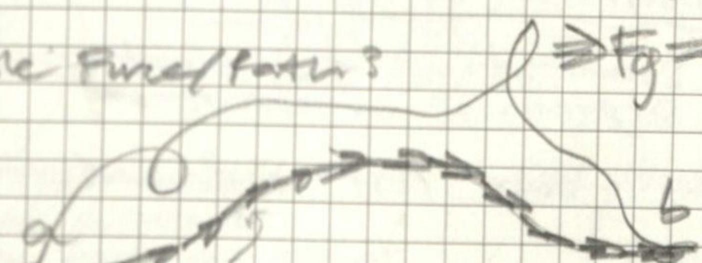
$$\Rightarrow dF_g = \frac{Gm(M/l)dx}{x^2}$$

$$\Rightarrow F_g = \int_{d}^{d+l} dF_g .$$

Ex. Constant Force:

$$W = \vec{F} \cdot \Delta\vec{r}$$

Variable Force/Path:



$$dW = \vec{F} \cdot d\vec{r}$$

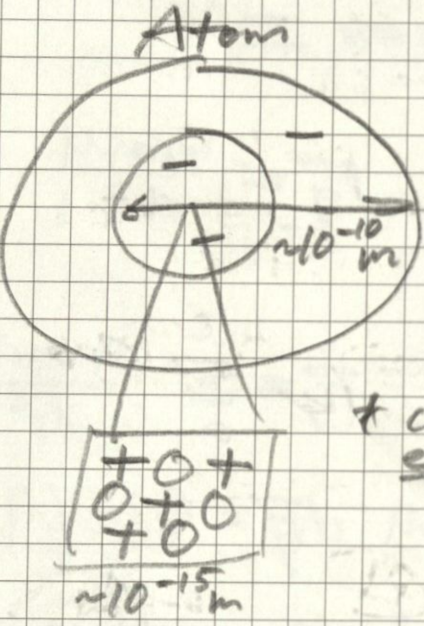
$$\Rightarrow W = \int_a^b \vec{F} \cdot d\vec{r}$$

Reading 01: 8.29.24

21.1: Electric Charge

* Items can be "charged" by being rubbed together
  * Two positive or negative charges repel each other
    A positive and a negative charge attract each other

The Structure of Matter:



Proton: positive charge
Mass $= 1.673 \cdot 10^{-27}$ kg

Neutron: No charge
Mass $= 1.675 \cdot 10^{-27}$ kg

Electron: Negative charge
Mass $= 9.109 \cdot 10^{-31}$ kg

* charges of proton and electron are <u>equal</u> in <u>magnitude</u>

<u>Atomic Number</u>: # of protons or electrons

<u>Positive Ion</u>: atom with removed electrons

<u>Negative Ion</u>: atom w/ additional electrons

<u>Ionization</u>: gain or loss of electrons

* When protons = electrons, atom is <u>neutral</u> and the object is electrically neutral

<u>Principle of Conservation of Charge</u>:

the Algebraic sum of all the electric charges of a closed system is constant

* No electric charge is ever lost or gained — only transferred
* The magnitude of charge of the electron or protons is a natural unit of charge

21.2: Conductors, Insulators, and Induced Charges

- some materials permit electric charge to transfer easily and others do not

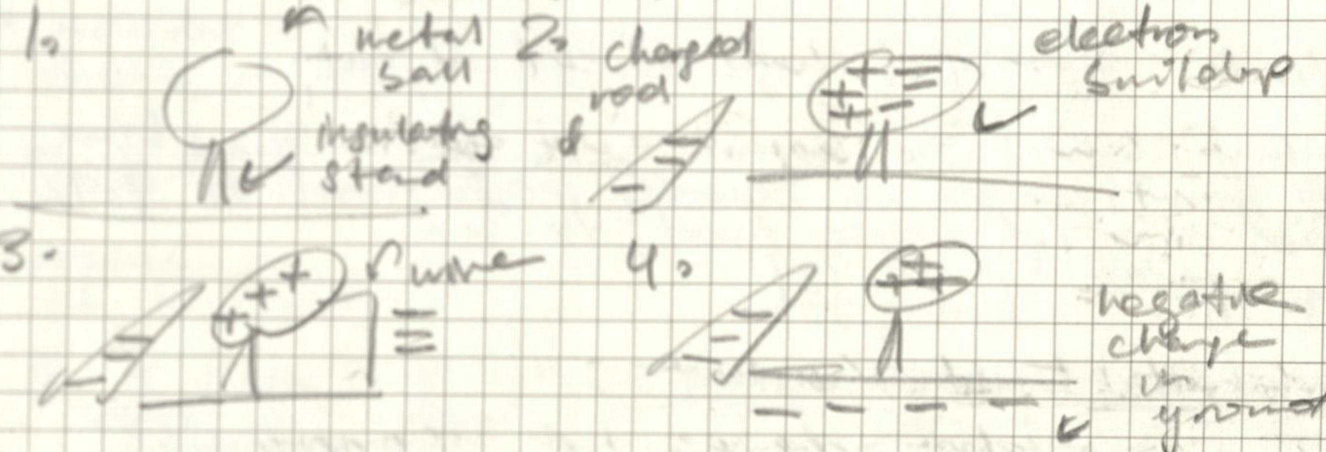
Conductor: materials that permit easy movement of charges

ex: copper wire, metals

Insulator: material that does not . . .

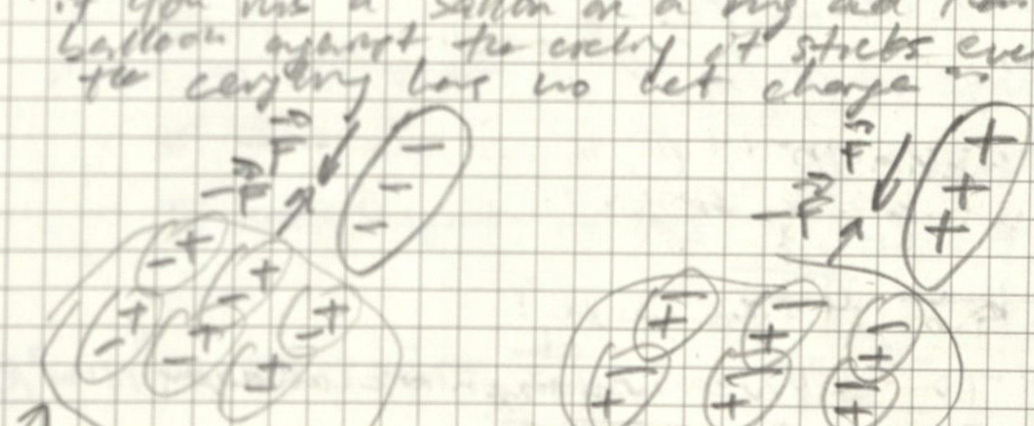
ex: rubber band, nylon thread, nonmetals

Charging by Induction: a technique where a charged object gives another object its opposite charge without losing any of its own charge



Electric Forces on Uncharged Objects:

"if you rub a balloon on a rug and then hold the balloon against the ceiling it sticks even though the ceiling has no net charge"



electrons in each molecule of the neutral insulator shift away from the object

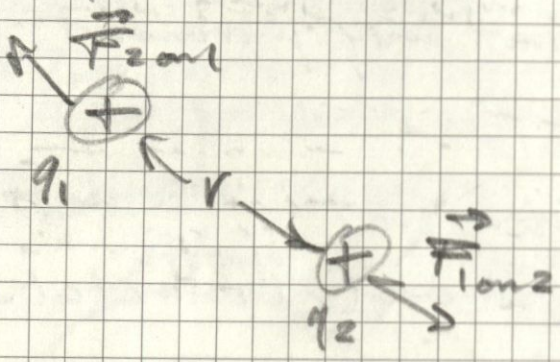
Polarization

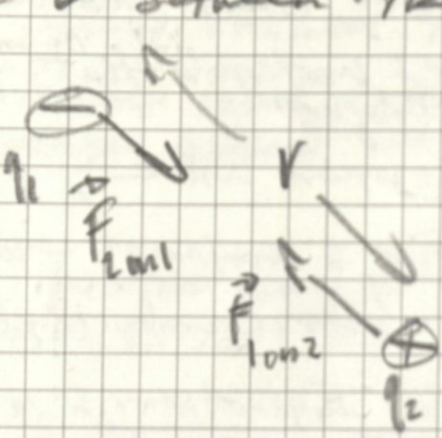
vice versa

attractive net forces via induction

21.3: Coulomb's Law

__Point Charges__: charged objects that are very small compared to the distance r between them





$$\vec{F}_{1 \text{ on } 2} = -\vec{F}_{2 \text{ on } 1}$$

$$F_{1 \text{ on } 2} = F_{2 \text{ on } 1} = k\frac{|q_1 q_2|}{r^2}$$

q (or Q): quantity of charge of each object

__Coulomb's Law__: The magnitude of electric force b/w 2 point charges is directly proportional to $q_1 q_2$ and inversely proportional to r^2

* $F = k\frac{|q_1 q_2|}{r^2}$

__Fundamental Electric Constants__:

__SI__ unit of electric charge: $1\,C$ "1 coulomb"

$k = 8.9875517 87 \cdot 10^9 N \cdot m^2 / C^2$

* $\cong 8.988 \cdot 10^9 N \cdot m^2 / C^2$

__Magnitude of charge of p^+/e^-__: in terms of coulombs

* $e = 1.602176634 \cdot 10^{-19} C$

__Electric Constant__: ε_0 "epsilon-naught (zero)"

* __SI__ unit of $k = \frac{1}{4\pi\varepsilon_0}$

$\varepsilon_0 = 8.854 \cdot 10^{-12} C^2/N \cdot m^2$, $\frac{1}{4\pi\varepsilon_0} = k = 8.99 \cdot 10^9 N \cdot m^2 / C$

* $\cong \frac{1}{4\pi\varepsilon_0} = 9.0 \cdot 10^9 N \cdot m^2 / C^2$

* $F = \frac{1}{4\pi\varepsilon_0}\frac{|q_1 q_2|}{r^2}$ ↖ values of 2 charges

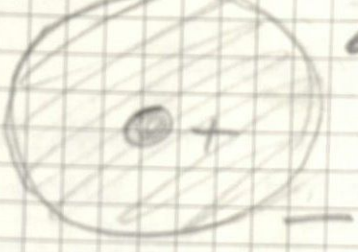
← distance b/w charges

coulomb's law

electric constant

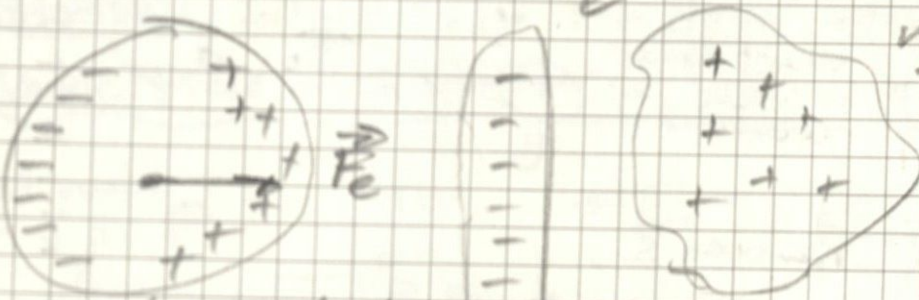
* Can also undergo superposition by vector sum *

Lecture 02: 8.30.24



atoms stay in balance by containing both positive and negative charges

fur gives its negative charges to the plastic rod



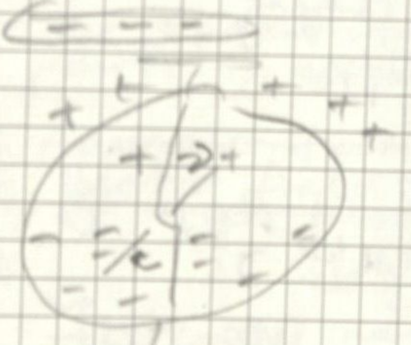
conductor insulator

* conductors allow 1-2 electrons per atom to float freely in the object

$F_e = \frac{kqQ}{r^2}$ very similar to $F_g = \frac{GmM}{r^2}$

* positive charge has a more powerful force since it is closer

Q. A charged rod is near the electroscope but does not touch it. The opposite side of the electroscope is grounded and the rod is removed. What happens

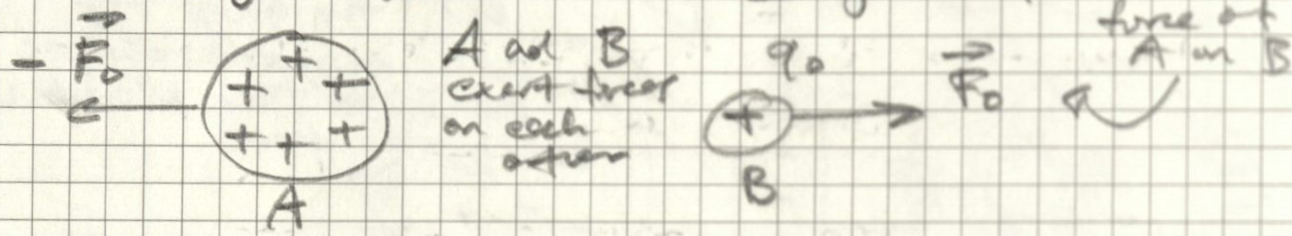


Reading 02: 9.1.24

21.4: Electric Field and Electric Forces

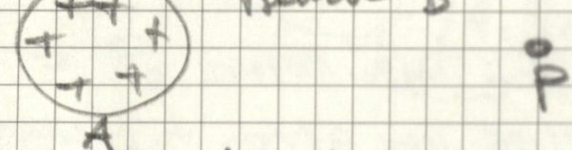
Electric Field: "action at a distance force"

- a force that acts across empty space without needing physical contact ex: gravity

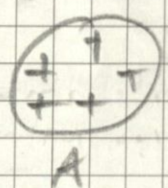


A and B exert forces on each other

force of A on B



Remove B



A sets up an electric field $\vec{E}$ at P

test charge q_0

$$\vec{E} = \frac{\vec{F}_0}{q_0}$$

force per unit of charge exerted by A on test charge at P

* the electric force on a charged object is exerted by the electric field created by other charged objects

$$\vec{E} = \frac{\vec{F}_0}{q_0}$$

$\vec{F}_0$: Electric force on test charge q_0 due to other charges

$\vec{E}$: electric field

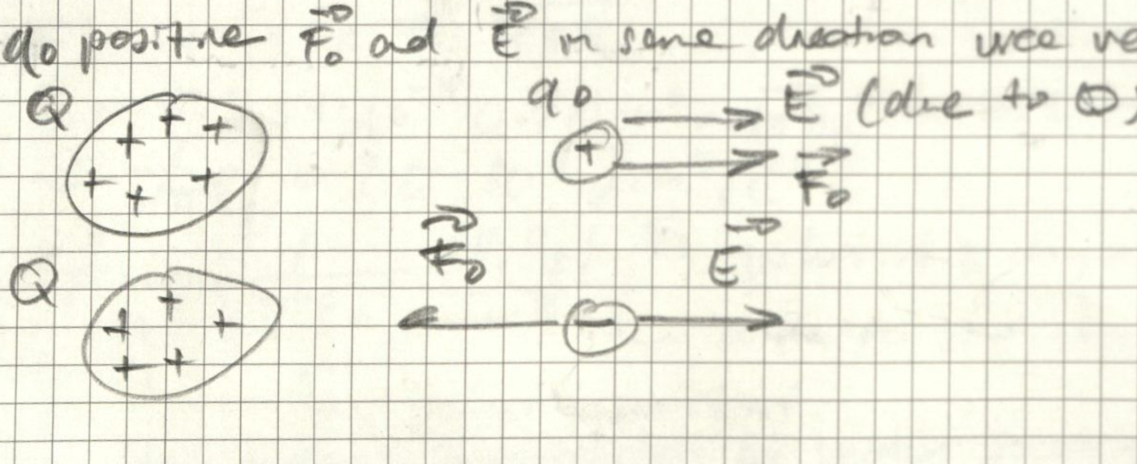
q_0: value of test charge

SI: 1 N/C

$$\vec{F}_0 = q_0 \vec{E}$$

(force exerted on a point charge q_0 by an electric field $\vec{E}$)

If q_0 positive $\vec{F}_0$ and $\vec{E}$ in same direction vice versa



Note: $\vec{F_0} = q_0\vec{E} \sim \vec{F_g} = m_0\vec{g}$

$\vec{E} = \frac{\vec{F_0}}{q_0}$ $\vec{g} = \frac{\vec{F_g}}{m_0}$

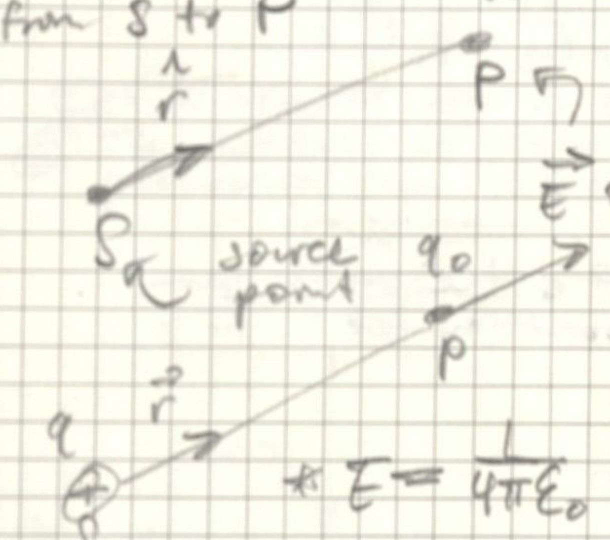
electric field gravitational field

CAUTION: $\vec{F_0} = q_0\vec{E}$ is for point test charges only

Electric Field of a Point Charge:

$* F_0 = \frac{1}{4\pi\varepsilon_0}\frac{|q q_0|}{r^2}$

unit vector from S to P



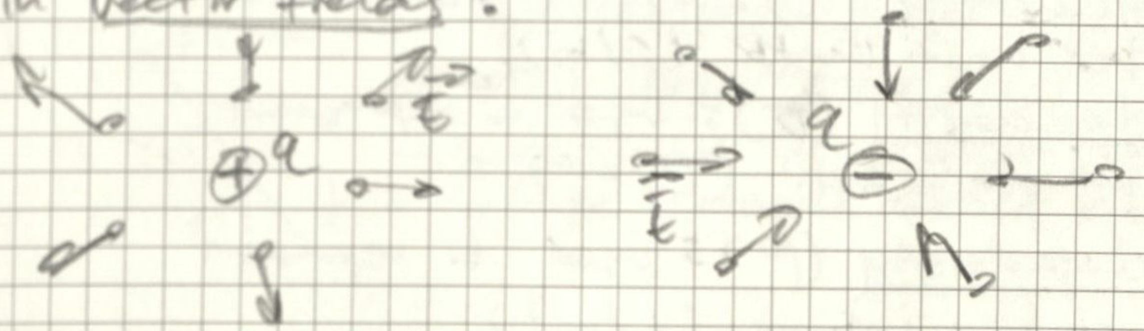
at each point P, $\vec{E}$ from q points away / towards charge in same / opposite direction of $\vec{r}$

$* E = \frac{1}{4\pi\varepsilon_0}\frac{|q|}{r^2}$ magnitude of $\vec{E}$ due to a point charge

$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{r}$ magnitude and direction

* charge is present whether or not there is a second point to experience the charge

In Application there are continuous and representation in vector fields:



* q produces an electric field at all points in space
* $\vec{E}$ at every point in a conductor must be 0
* positive $\vec{E}$ repel while negative $\vec{E}$ attract

21.5: Electric-Field Calculations

$\vec{E} = \frac{\vec{F}}{q_0}$ gives electric field of a point charge q_0

in most situations this charge is actually distributed over space

Superposition of Electric Fields:

imagine a charge distribution to be made up of $q_1, q_2, q_3, \ldots$

at any P, this produces $\vec{E}_1, \vec{E}_2, \vec{E}_3, \ldots$

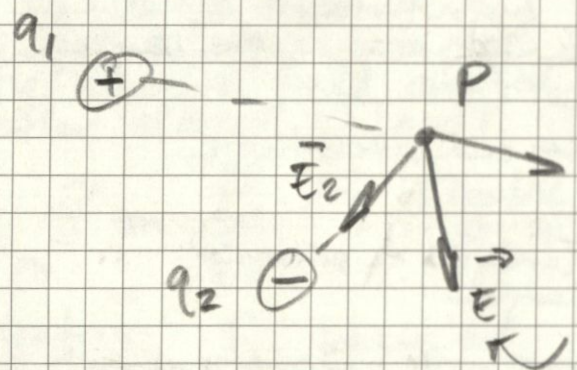
by the superposition principle this produces:

$$\vec{F_0} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \ldots = q_0\vec{E}_1 + q_0\vec{E}_2 + q_0\vec{E}_3 + \ldots$$

this combined effect gives:

$$\vec{E} = \frac{\vec{F}_0}{q_0} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + \ldots$$

ie, the total electric field at P is the vector sum of the fields at P due to each point in the charge distribution



total electric field at P by vector sum of $\vec{E}_1$, $\vec{E}_2$

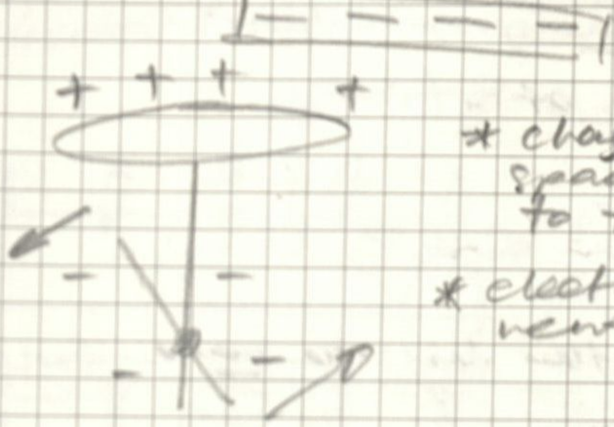
linear charge density (λ): used to represent charge per unit length (C/m)

surface charge density (σ): used to represent charge per unit area (C/m^2)

volume charge density (ρ): used to represent charge per unit volume (C/m^3)

Lecture 03: 9.2.24

electroscope: old-fashion analog device for measuring charge



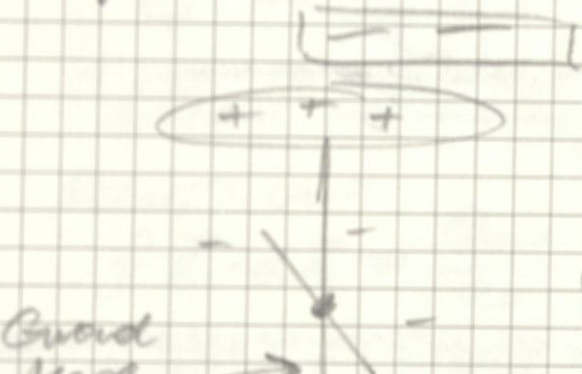
* charges cannot jump through space — must have contact to transfer charges

* electroscope starts electrically neutral

1

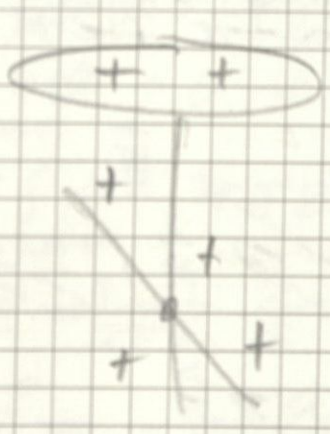
Ex. A charged rod is brought near the electroscope. 2 The bottom of the electroscope is then briefly grounded then the rod is removed. What happens?

3



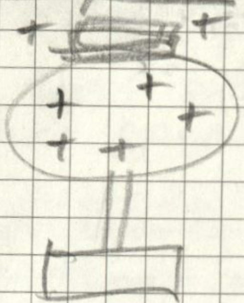
1. arm moves (reads nonzero)
2. moves back to zero
3. **returns** to nonzero

* cannot tell you what kind of charge is present



- negative charges leave via ground
- pos charges stay b/c of rod
- when rod is removed pos charge spreads across electroscope

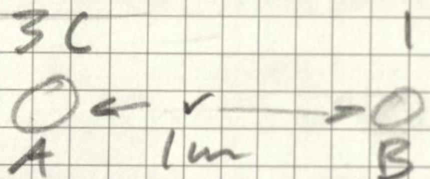
Vandegraph: builds up charge on the drum



Q. what will happen to the plates when the vandegraph turns on

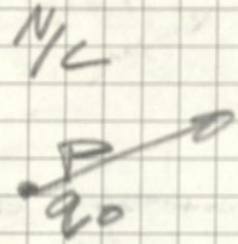
Fly off one at a time

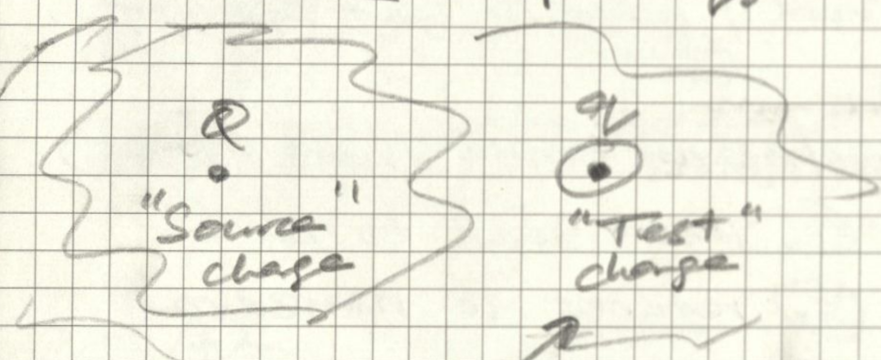
Ex. Which feels the larger force?

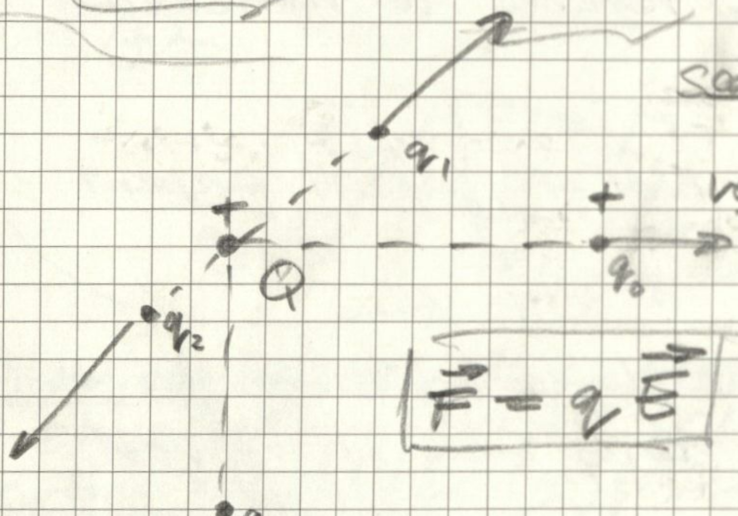


they feel the same force

Electric Field: $\vec{E}_P = \dfrac{\vec{F}_{\text{on } q_0}}{q_0}$ SI: N/C





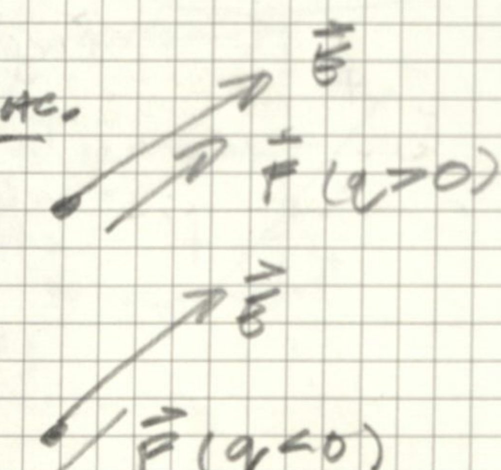


Scalar field: only magnitude
(ex. temperature fields)

vector field: magnitude and direction
(ex. electric field)

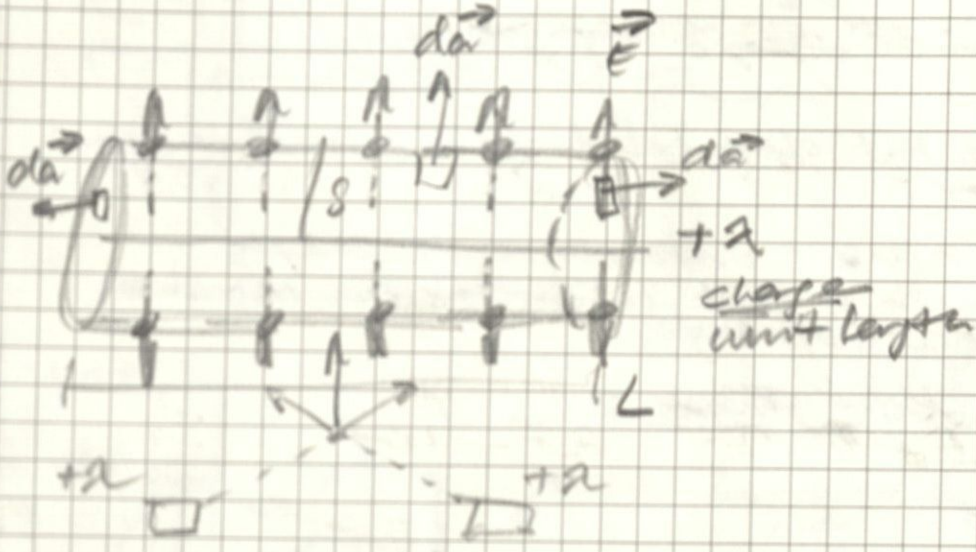
$$\boxed{\vec{F} = q\vec{E}}$$

Note.



RQ #4 Notes: 9.3.24

Ex - You have an infinite line of charge with an electric field strength of 800 N/C at a point 11 cm away. What is the line's linear charge velocity?



* symmetry cancels out all diagonal lines — only vertical ones!

Gauss's Law:

$$\oint \vec{E}\cdot d\vec{a} = \frac{Q_{enc}}{\varepsilon_0}$$

$$\int_{\text{sleeve}} \vec{E}\cdot d\vec{a} + \int_{\text{end caps}} \vec{E}\cdot d\vec{a} = \frac{Q_{enc}}{\varepsilon_0}$$

angle b/w $\vec{E}$ and $d\vec{a}$

$$\int E\,da\cos 0^\circ + \int E\,da\cos 90^\circ = \frac{Q_{enc}}{\varepsilon_0}$$

$$\int_{\text{sleeve}} E\,da + 0 = \frac{Q_{enc}}{\varepsilon_0}$$

$$E\int_{\text{sleeve}} da = E\,2\pi sL = \frac{\lambda L}{\varepsilon_0}$$

$$E = \frac{1}{2\pi\varepsilon_0}\frac{\lambda}{s}$$

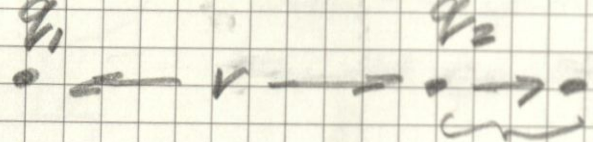
$$\vec{E} = \frac{1}{2\pi\varepsilon_0}\frac{\lambda}{s}\hat{s}$$

$$\Rightarrow \lambda = 2\pi\varepsilon_0 s E$$

Lecture 04: 09.09.24

Electric Field: $\boxed{\vec{F} = q\vec{E}}$

* each charge both produces and is affected by electric fields *

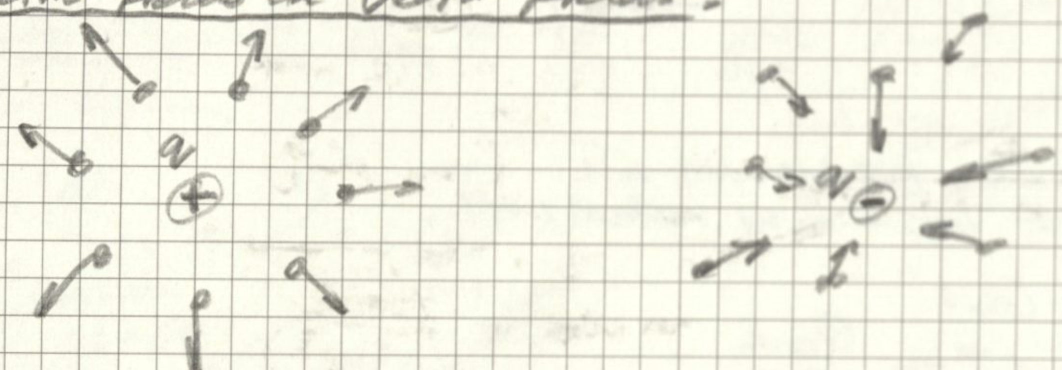


$$F = \frac{k|q_1 q_2|}{r^2}$$

Coulomb's law does not obey relativity — cannot explain dynamic particles

Electric fields "ripple" with dynamic charges like a puddle with a rock thrown at it — can explain dynamic particles

Electric Fields are Vector Fields:



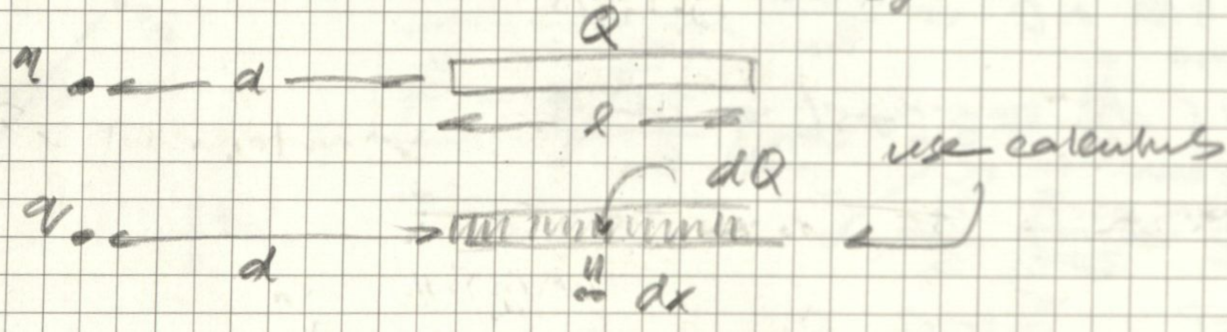
Continuous Charge Distributions:

a typical charge is ~μC, which is about

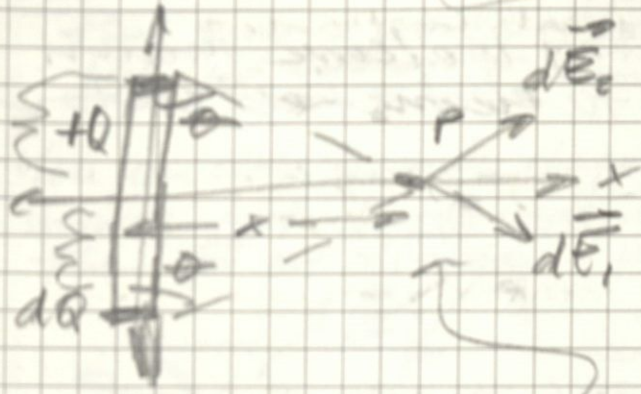
$$\frac{1\cdot 10^{-6}C}{1.6\cdot 10^{-19}C} \approx 6\cdot 10^{12} \text{ electrons!}$$

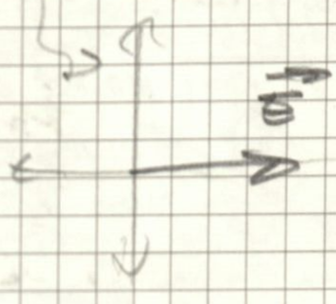
↳ charge of an electrons

So we treat it as a continuous charge:

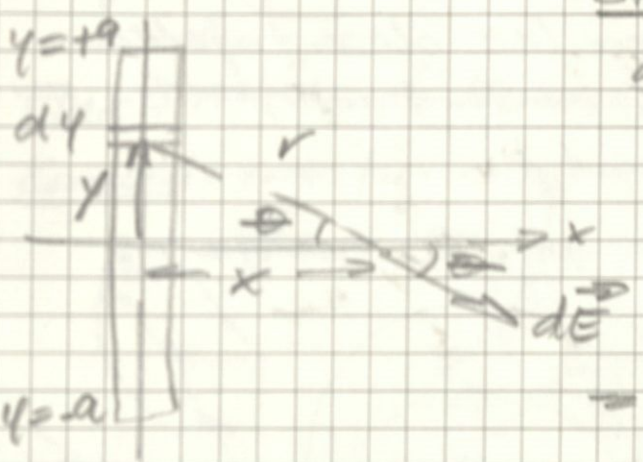


Ex: What is the direction of the net electric field at point P. What is the magnitude?





pairwise cancelations



Start:

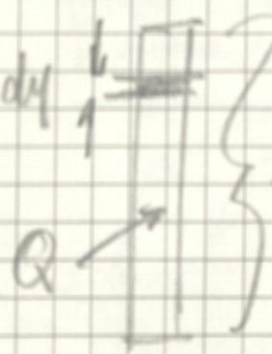
$$dE = \frac{k\,dQ}{r^2}$$

but want to write in terms of

$$\int \dots dy$$

$$\Rightarrow r^2 = x^2 + y^2$$

$$\Rightarrow dE = \frac{k\,dQ}{(x^2+y^2)}$$

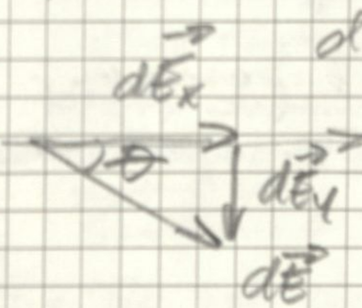


$$\frac{dQ}{Q} = \frac{dy}{\ell} \Rightarrow dQ = \frac{Q}{\ell}\,dy$$

$$\Rightarrow dE = \frac{k\frac{Q}{\ell}\,dy}{(x^2+y^2)}$$

$$dE_x = dE\cos\theta$$

→ θ in terms of y?



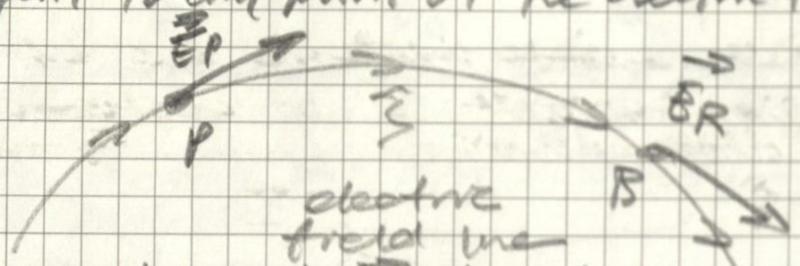
$$\cos\theta = \frac{x}{\sqrt{x^2+y^2}}$$

$$dE_x = \frac{k(Q/\ell)\,dy}{(x^2+y^2)}\,\frac{x}{\sqrt{x^2+y^2}}$$

Reading 05: 9.6.24

21.6: Electric Field Lines

<u>electric field line</u>: an imaginary line/curve drawn tangent to any point of the electric field vector



Shows <u>direction</u> of $\vec{E}$ at each point

Also shows <u>magnitude</u>:

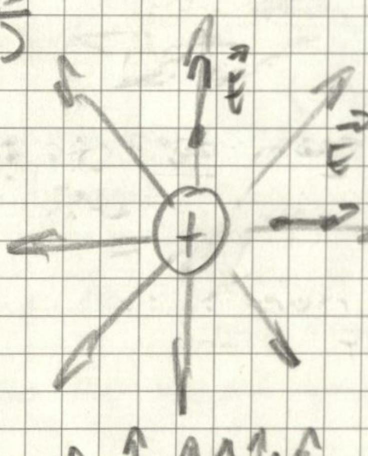
- where $\vec{E}$ is strong, we draw lines close together
- where $\vec{E}$ is weak, lines are farther apart

* field lines <u>never</u> intersect

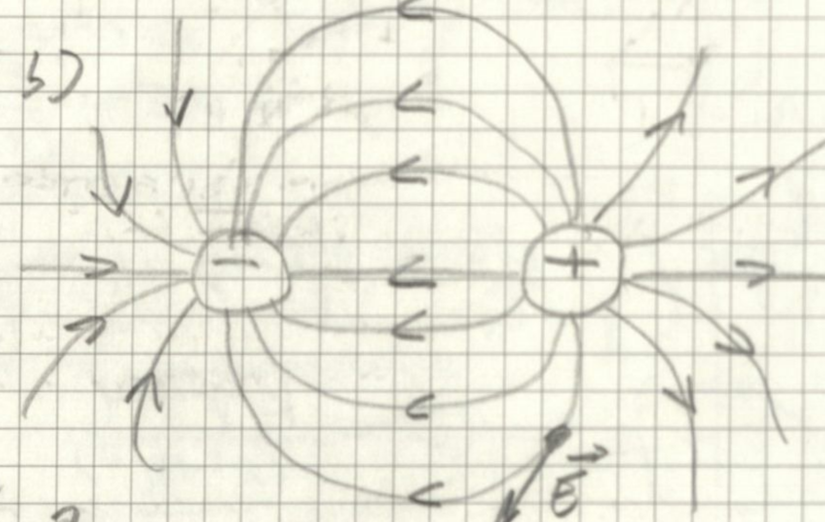
<u>Field Maps</u>: 2D cross section of 3D patterns

<u>Ex:</u>

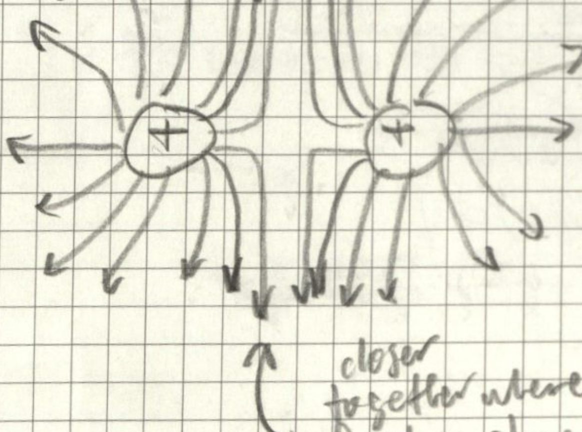
a)



b)



c)

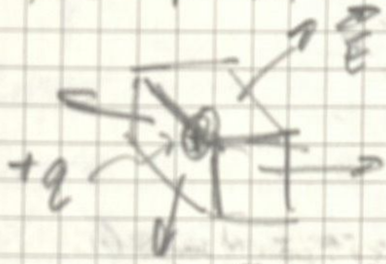


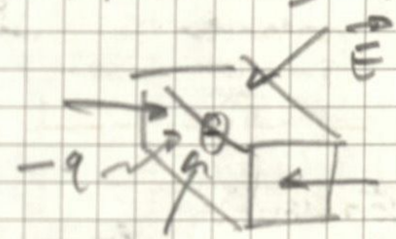
<u>CAUTION</u>: electric field lines are <u>not</u> trajectories

22.1: Charge and Electric Flux

closed surface: imaginary surface enclosing a volume that may or may not have charge inside and yet has no affect on the surrounding electric field

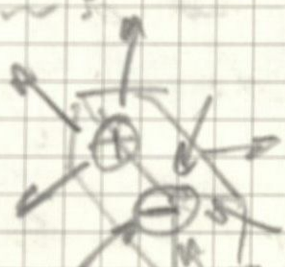
You can determine the charge inside the box by moving test charges outside the box and measuring the $\vec{E}$ at different positions using $\vec{E} = \vec{F}/q_0$:

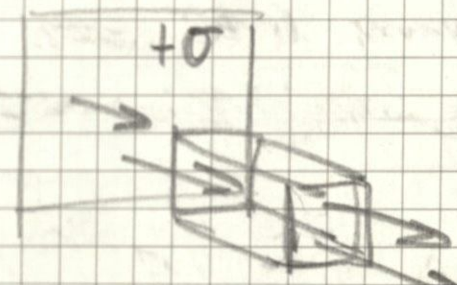




Flux: net "fluid" flow escaping a surface

Kernerel: net flow:





zero net charge inside box | zero net charge inside box | zero net charge inside box

inward flux cancels outward flux

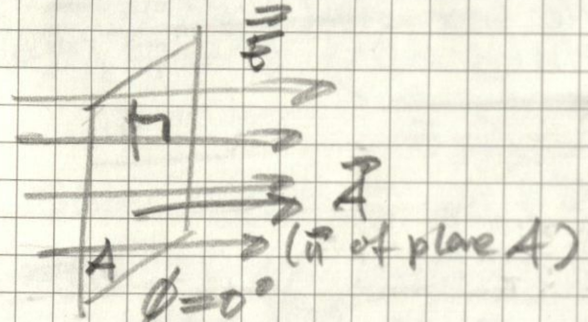
Summary:

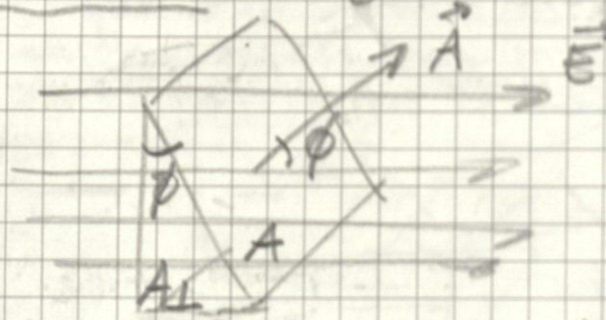
1. whether there is net inward or outward electric flux through a closed surface depends on the sign of the enclosed charge
2. charges outside the closed surface do not give a net electric charge through the surface
3. The net electric charge is proportional to the net amount of charge enclosed within the surface but is otherwise independent of the size of the closed surface

22.2: Calculating Electric Flux

Gauss's law: the net electric flux through a closed surface is directly proportional to the net charge inside the surface

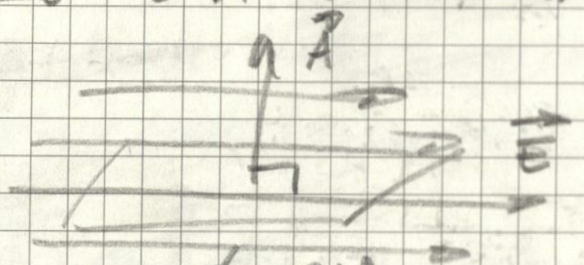
Flux of a Uniform Electric Field: $\Phi_E = \vec{E} \cdot \vec{A}$





$$\Phi_E = \vec{E} \cdot \vec{A} = EA\cos\phi = E_\perp A = A_\perp E$$

$$\Phi_E = \vec{E} \cdot \vec{A} = EA\cos 0° = EA$$



* $\vec{A} = A\hat{n}$

* $\hat{n}$ always outward (positive) vice versa

$\phi = 90°$

$$\Phi_E = \vec{E} \cdot \vec{A} = EA\cos 90° = 0$$

Flux of a Nonuniform Electric Field: $d\vec{A} = \hat{n}\, dA$

if $\vec{E}$ varies from point to point
if A is part of a curved surface

$$\Phi_E = \int E\cos\phi\, dA = \int E_\perp\, dA = \int \vec{E} \cdot d\vec{A}$$

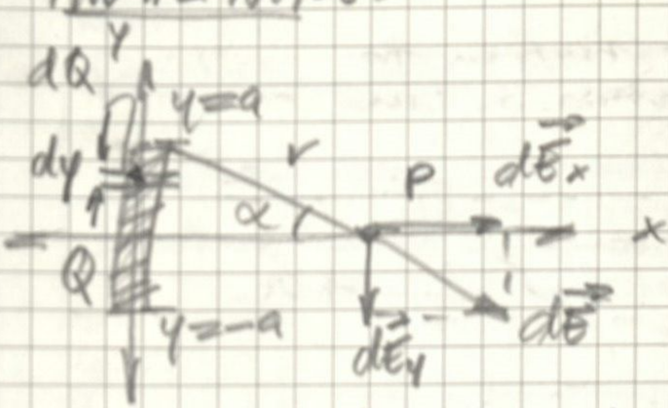
electric flux through surface
angle b/w $\vec{E}$ and $\vec{A}$
element of surface area
vector element of surface area

* Think: surface integral

HW#2 Notes:

9.8.24

Find $\vec{E}$ at point P.



Linear charge density: $\lambda = \frac{Q}{2a}$

Charge in 1-segment: $dQ = \lambda dy = \frac{Q}{2a} dy$

distance: $r = (x^2+y^2)^{1/2}$

From $E = \frac{kQ}{r^2} \sim dE = \frac{k\,dQ}{r^2}$; $\left(\begin{array}{l} \cos\alpha = \frac{x}{r} \\ \sin\alpha = \frac{y}{r} \end{array}\right)$

$$dE_x = dE\cos\alpha = \frac{1}{4\pi\varepsilon_0}\frac{Q}{2a}\frac{x\,dy}{(x^2+y^2)^{3/2}}$$

$$dE_y = -dE\sin\alpha = -\frac{1}{4\pi\varepsilon_0}\frac{Q}{2a}\frac{y\,dy}{(x^2+y^2)^{3/2}}$$

Integrate from $y=-a$ to $y=a$:

$$E_x = \frac{1}{4\pi\varepsilon_0}\frac{Q}{2a}\int_{-a}^{a}\frac{x\,dy}{(x^2+y^2)^{3/2}} \stackrel{?}{=} \frac{Q}{4\pi\varepsilon_0}\frac{1}{x\sqrt{x^2+a^2}}$$

$$E_y = \frac{1}{4\pi\varepsilon_0}\frac{Q}{2a}\int_{-a}^{a}\frac{y\,dy}{(x^2+y^2)^{3/2}} = 0 \text{ by symmetry}$$

Vector Form:

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{x\sqrt{x^2+a^2}}\hat{i}$$

NOTE: $\vec{E}$ points away from line of charge if λ is positive and towards the line of charge if λ is negative

Reading Notes: 9.8.24

22.3: Gauss's Law: alternative to Coulomb's Law

Point Charge Inside a Spherical Surface:

"the total electric flux through a closed surface is proportional to the total (net) electric charge inside the surface"

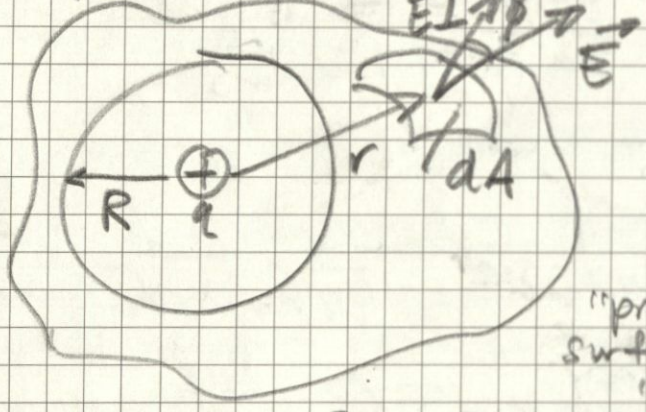
$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{R^2} \leftarrow \text{sphere of radius } R$$

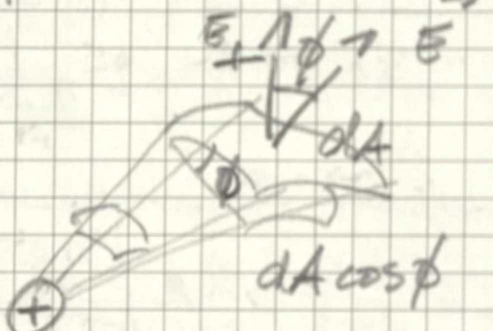
$$\Phi_E = EA = \frac{1}{4\pi\varepsilon_0}\frac{q}{R^2}(4\pi R^2) = \frac{q}{\varepsilon_0}$$

"flux is independent of R, only dependent on charge of contained inside R"

Point Charge Inside Nonspherical Surface:

"can project any irregular surface onto a sphere and still use law"





"projection of dA of irregular surface onto spherical surface is $dA\cos\phi$"

$$\Phi_E = \underbrace{\oint \vec{E}\cdot d\vec{A}}_{\text{electric flux through closed surface of area } A \;=\; \text{surface integral of } \vec{E}} = \frac{Q_{encl}}{\varepsilon_0} \leftarrow \text{total charge enclosed by surface}$$

Other Forms:

$$\Phi_E = \oint E\cos\underbrace{\phi}_{\text{angle b/w } \vec{E} \text{ and } \hat{n}} dA = \oint E_\perp dA = \oint \vec{E}\cdot d\vec{A} = \frac{Q_{encl}}{\varepsilon_0}$$

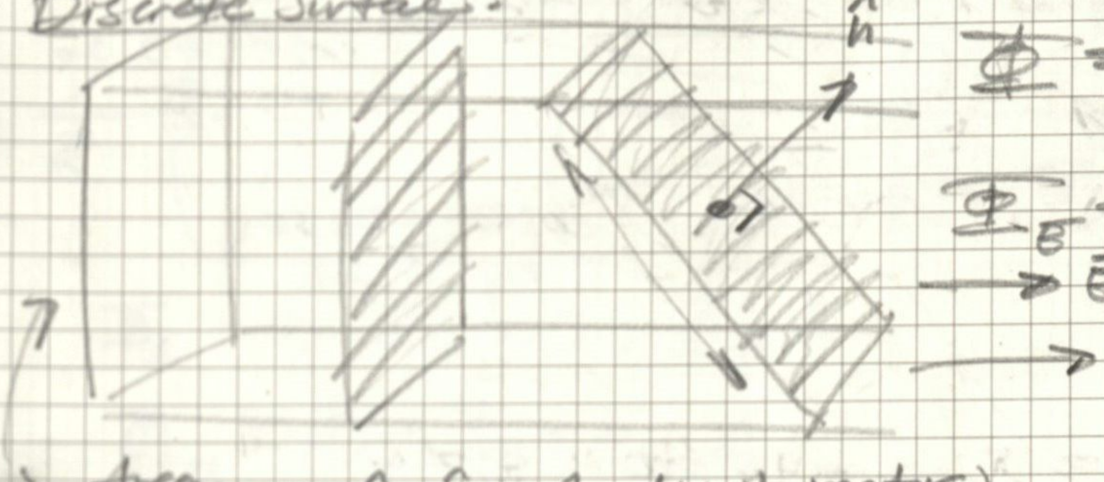
★ Examples in 22.5: Applications of Gauss's Law ★

outward flux → positive charge

inward flux → negative charge

Lecture Notes: 9.9.24

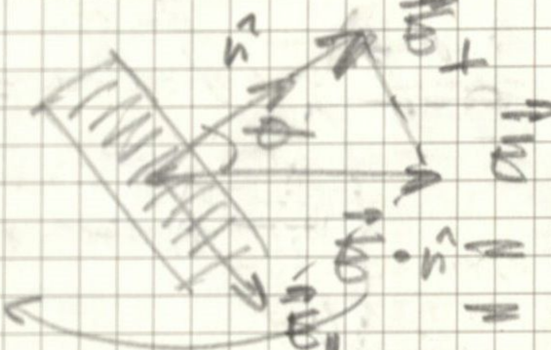
Discrete Surface:



$\Phi = Av$

$\Phi_E = AE$

$\hat{n} \cdot \hat{n} = 1$ (unit vector)



$\Phi = \vec{v} \cdot \hat{n} A$

$$\boxed{\Phi_E = \vec{E} \cdot \hat{n} A}$$

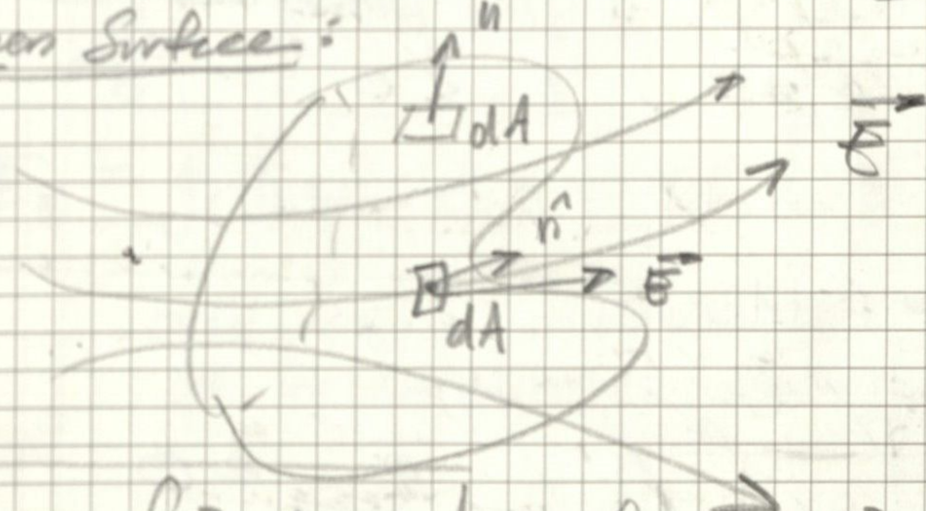
$= EA\cos\theta$

$\vec{E} \cdot \hat{n} = E n \cos\theta$ ($n \to 1$)

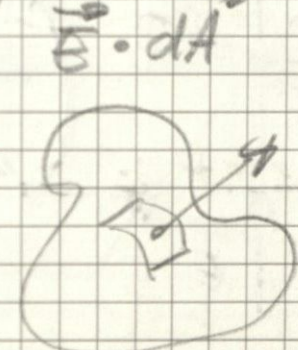
$= E\cos\theta$

$= E_\perp$

Open Surface:



$$\boxed{\Phi_E = \int_S \vec{E} \cdot \hat{n}\, dA} = \int_S \vec{E} \cdot d\vec{A}$$



$\hat{n}$ outwards facing normals

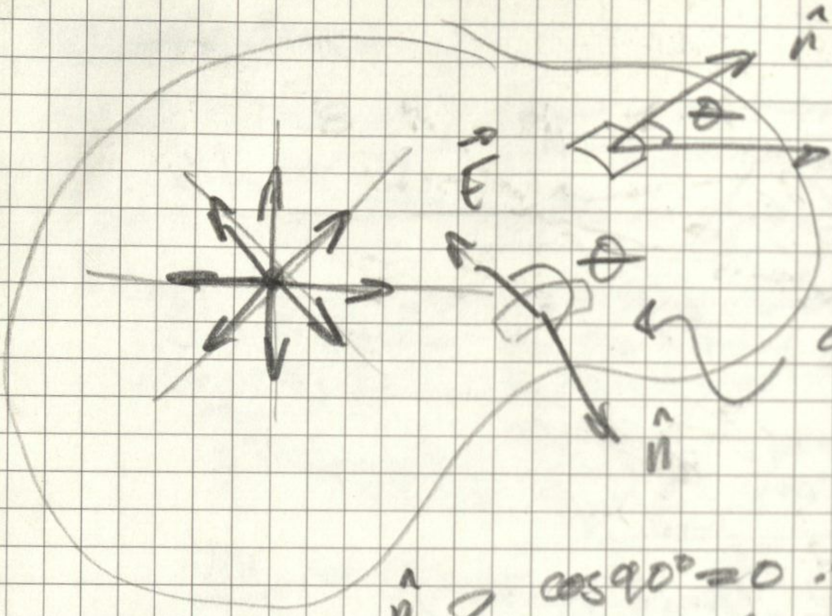
Closed Surface:

$\Phi_E = \oint_S \vec{E} \cdot \hat{n}\, dA$

Φ_E "measure" of # field lines crossing surface

outwards - positive

inwards - negative

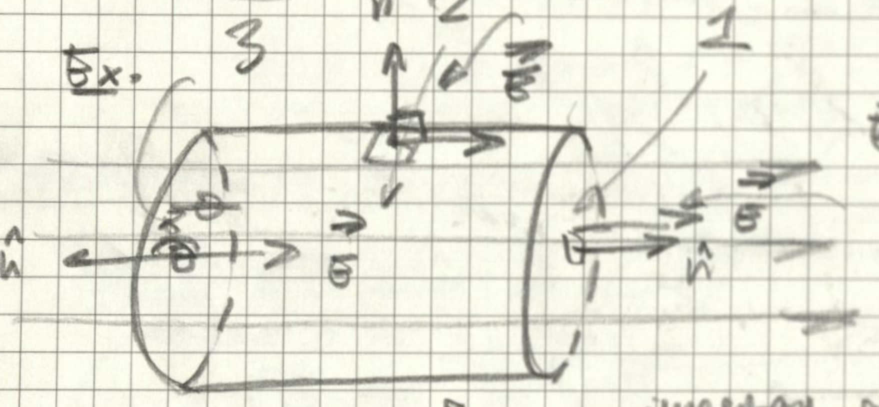


$\cos\theta > 0$ so pos Φ

$\cos\theta < 0$ so neg Φ_E

$\cos 90° = 0 \therefore \Phi = 0$

Ex.



$\vec{E}$ (uniform)

$d\Phi_E = +EdA$
in same direction

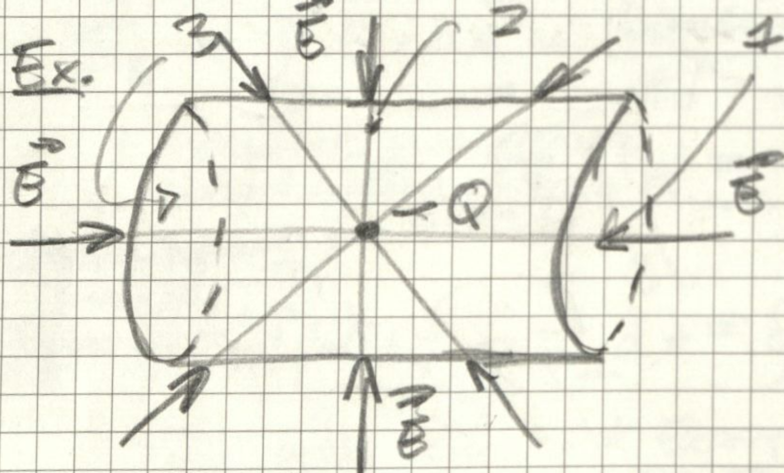
imaginary surface

$\Phi_1 > 0$, $\Phi_2 = 0$, $\Phi_3 < 0$

$\Phi_E = \oint_S \vec{E} \cdot \hat{n}\, dA$ ($\hat{n}\, dA = d\vec{A}$)

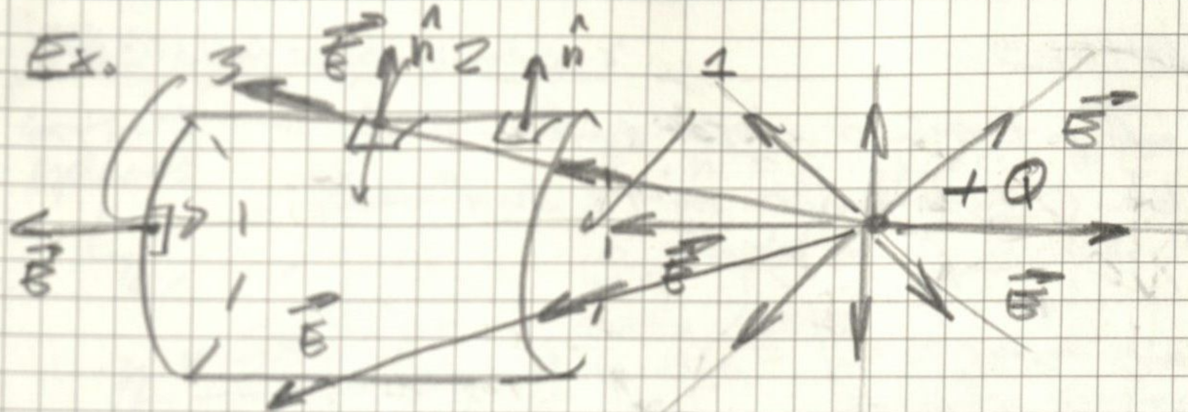
$\vec{E} \cdot \hat{n}\, dA = E\, dA \cos\theta$

Ex.



$\Phi_{1,2,3} < 0$

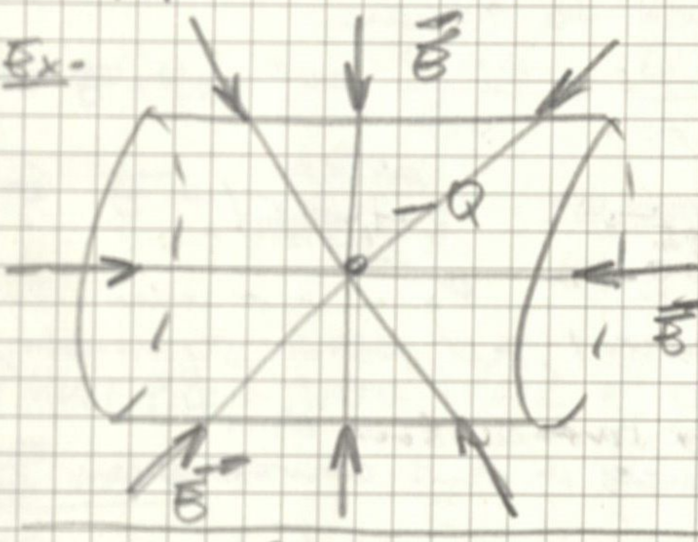
Ex.



$\Phi_1 < 0, \ \Phi_2 > 0, \ \Phi_3 > 0$

$\Phi_{net} = \Phi_1 + \Phi_2 + \Phi_3 = 0$

Ex.



$\Phi_{net} < 0$

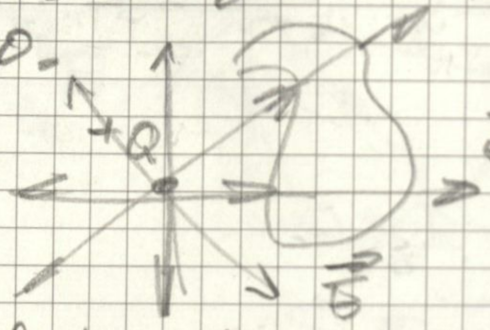
$$\boxed{\Phi_E = \oint \vec{E} \cdot \hat{n} \, dA = \frac{Q_{encl}}{\varepsilon_0}}$$

only way to get nonzero Φ is by having charge inside a surface

Lecture Notes: 9.11-24

Ex. If Φ_E through a closed surface is 0, must $\vec{E}$ be 0 everywhere on the surface?

NO.



* all field lines that enter the surface from outside the surface must exit the surface

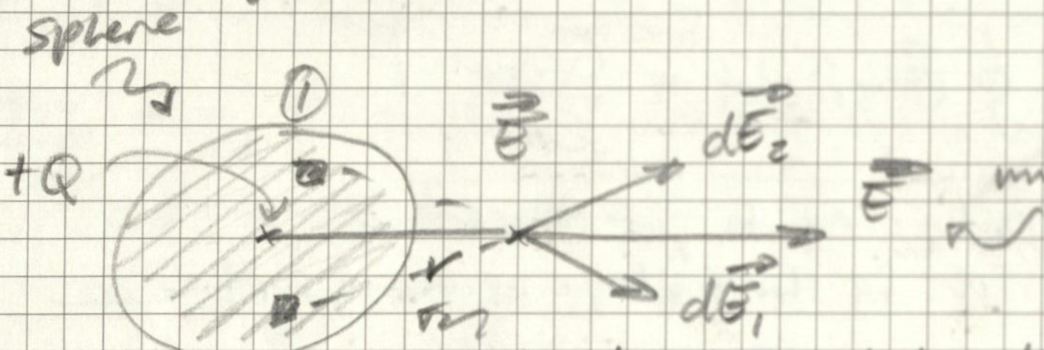
Coulomb: $E = \frac{k|Q|}{r^2}$

Gauss: $\oint \vec{E} \cdot d\vec{A} = \frac{Q_{encl}}{\varepsilon_0}$ study these!

Surfaces: spherical, cylindrical, and planar symmetry

(Gauss's law applies to any surface but will only be dealing with 3-cases in this course)

Ex.



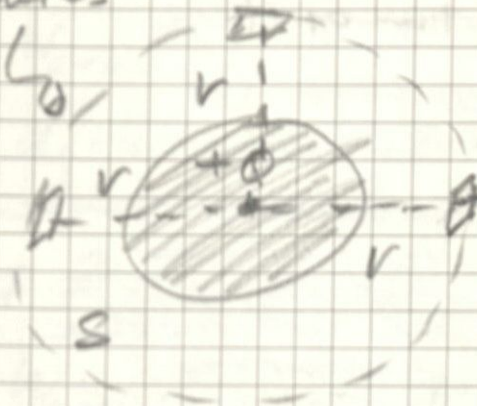
no matter how you rotate the sphere $\vec{E}$ will look like this!

Trick: choose the surface S so that E must be constant over it

* lots of symmetry involved here

Pick a Gaussian Surface:

both spheres



$$\vec{E}\cdot\hat{n} = E(1)\cos 0^\circ = E$$

$$\oint_S \vec{E}\cdot d\vec{A} = \oint_S E\,dA$$

$\hat{n}$ $\vec{E}$ * E is constant over the surface *

$$\therefore E\underbrace{\oint_S dA}_{\text{Surface Area of } S} = E\cdot 4\pi r^2$$

$$\Rightarrow E\cdot 4\pi r^2 = \frac{Q_{enc}}{\varepsilon_0} = \frac{Q}{\varepsilon_0}$$

$$\Rightarrow E = \frac{Q}{4\pi\varepsilon_0 r^2}$$

remember: $k = \frac{1}{4\pi\varepsilon_0}$

* but won't be useful much anymore

* Must pick a surface where $\vec{E}$ is constant over the surface! *

$\vec{E}$ inside:

* Gaussian surface can be inside charge!

Inside:

$$E\cdot 4\pi r^2 = \frac{Q_{encl}}{\varepsilon_0}$$

$$Q_{enc} = Q\,\frac{\frac{4}{3}\pi r^3}{\frac{4}{3}\pi R^3}$$

$$\Longleftrightarrow \frac{Q}{\frac{4}{3}\pi R^3} = \frac{Q_{encl}}{\frac{4}{3}\pi r^3}$$

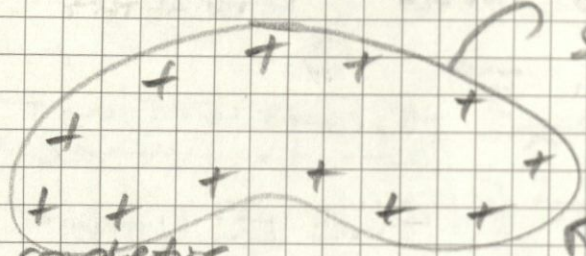
$$E\cdot 4\pi r^2 = \frac{Q r^3}{\varepsilon_0 R^3}$$

$$E_{in} = \frac{Q r}{4\pi\varepsilon_0 R^3}$$

Reading Notes: 9.12.24

22.5 - Charge on Conductors

Electrostatic Situation: no net motion of charge

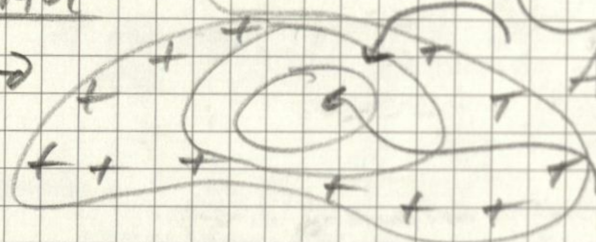


Solid conductor w/ charge q_c

charge resides on surface of the conductor

$\vec{E} = \vec{0}$ within the conductor

Same conductor w/ cavity



Arbitrary Gaussian Surface A

Cavity

$q_c + q$

For $\vec{E}$ to be 0 on all points of the Gaussian surface, the surface of the cavity must have a total charge $-q$

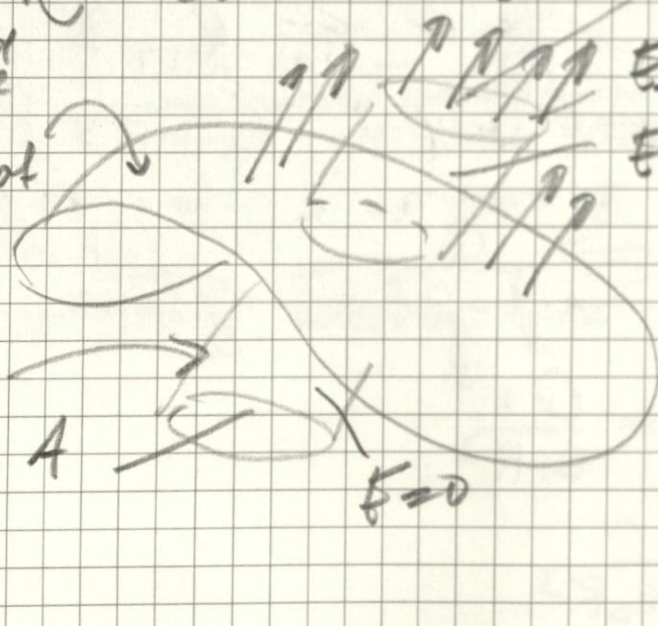
Field at the Surface of a Conductor:

$$E_\perp = \frac{\sigma}{\epsilon_0}$$

← surface charge density

electric constant A

Electric Field at surface of conductor



$E_\perp = E$

$E_\perp = 0$

Outer surface of charged conductor

Gaussian Surface

A

$E=0$

Reading Notes: 9.15.24

23.1: Electric Potential Energy

$$W_{a\to b} = \int_a^b \vec{F}\cdot d\vec{l} = \int_a^b F\cos\phi \, dl$$ (work done by a force)

$d\vec{l}$: infinitesimal distance on path

ϕ: angle btw $\vec{F}$ and $d\vec{l}$ at each point

If $\vec{F}$ is conservative: work done is independent of path

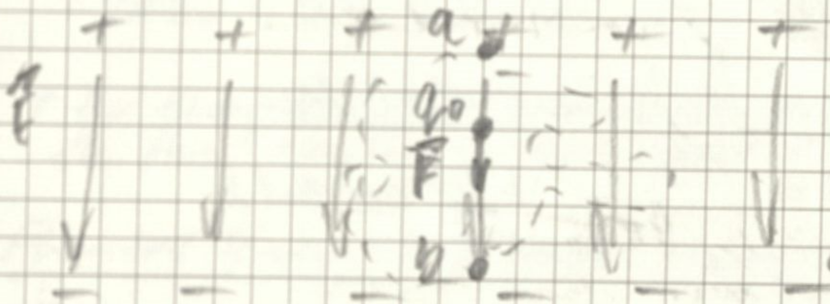
$$W_{a\to b} = U_a - U_b = -(U_b - U_a) = -\Delta U$$

* when work is done ($W_{a\to b} > 0$), potential energy decreases

$$K_a + U_a = K_b + U_b$$ (total mechanical energy is conserved)

Electric Potential Energy in a Uniform Field

$$W_{a\to b} = Fd = q_0 E d$$



* independent of path

aka conservative

$F_y = -q_0 E$ ~ $F_y = -mg$

$U = q_0 E y$ ~ $U = mgy$

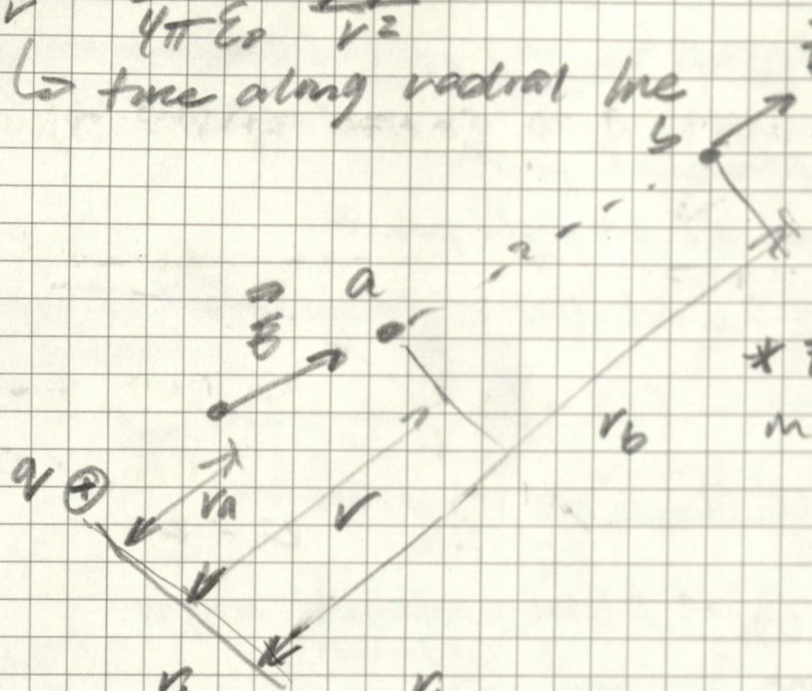
When q_0 moving from y_a to y_b:

$$W_{a\to b} = -\Delta U = -(U_b - U_a) = \dots = q_0 E (y_a - y_b)$$
$$= -q_0 E \Delta y$$

Electric Potential Energy of Two Point Charges:

$$F_r = \frac{1}{4\pi\varepsilon_0}\frac{q q_0}{r^2}$$

↳ force along radial line



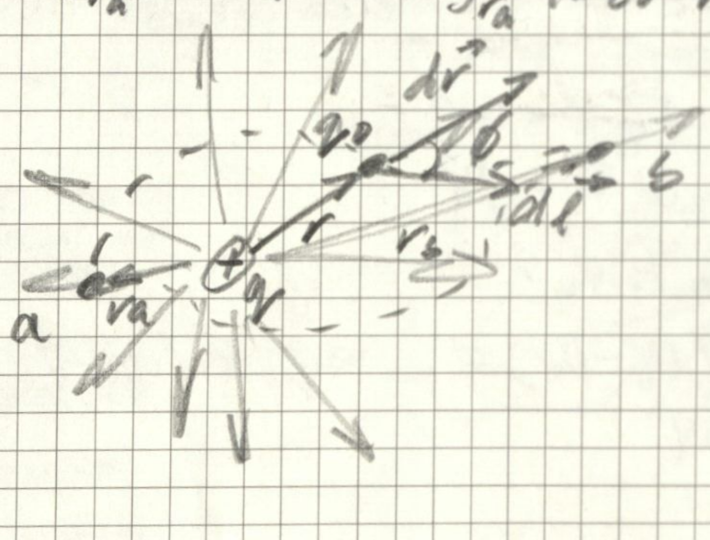
* F_r is not constant in this case

$$W_{a\to b} = \int_{r_a}^{r_b} F_r\,dr = \int_{r_a}^{r_b} \frac{1}{4\pi\varepsilon_0}\frac{q q_0}{r^2}\,dr = \frac{q q_0}{4\pi\varepsilon_0}\left(\frac{1}{r_a} - \frac{1}{r_b}\right)$$

* independent of path / conservative *

If a and b do not lie on the same radial line:

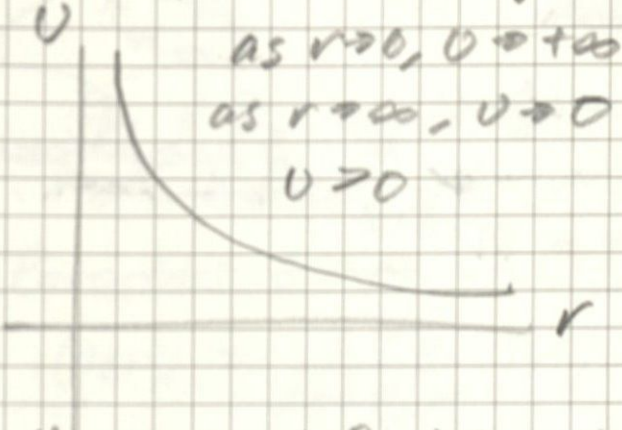
$$W_{a\to b} = \int_{r_a}^{r_b} F\cos\phi\,dl = \int_{r_a}^{r_b} \frac{1}{4\pi\varepsilon_0}\frac{q q_0}{r^2}\cos\phi\,dl$$

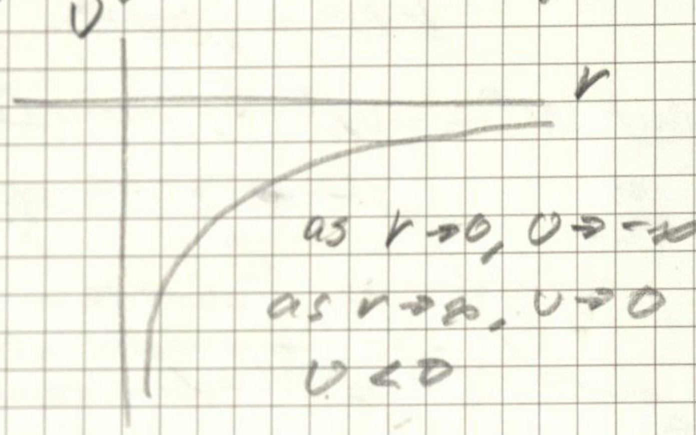


Electric Potential Energy:

$$U = \frac{1}{4\pi\varepsilon_0}\frac{q q_0}{r}$$

q and q_0 have same sign: q and q_0 have opposite signs





U with Several Point Charges:

$\vec{E}$ is a vector sum
U is an algebraic sum

$$U = \frac{q_0}{4\pi\varepsilon_0}\left(\frac{q_1}{r_1} + \frac{q_2}{r_2} + \frac{q_3}{r_3} + \ldots\right) = \frac{q_0}{4\pi\varepsilon_0}\sum_i \frac{q_i}{r_i}$$

* for every electric field due to a static charge distribution, the force exerted by that field is conservative

Potential Energy of Interacting Charges?

$$U = \frac{1}{4\pi\varepsilon_0}[c]\sum_{i<j}\frac{q_i q_j}{r_{ij}} \quad ?$$

Lecture Notes: 9.16.24

Potential Energy:

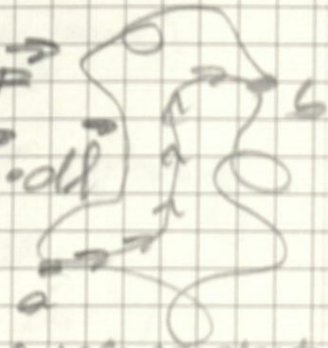
spring: $F_x = -kx$

$\Rightarrow U(x) = \frac{1}{2}kx^2$

$K(x) = \frac{1}{2}mv^2$

Work done by $\vec{F}$

$$W^F_{a\to b} = \int_a^b \vec{F}\cdot d\vec{l}$$

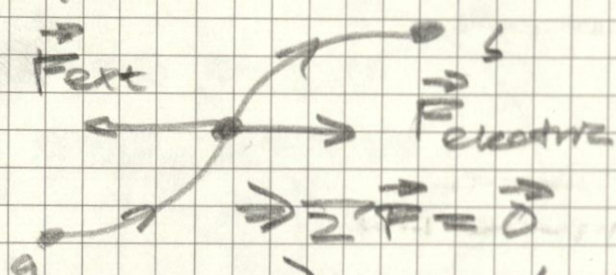


Why $W^F = -W^{Ext}$?

$\Delta U = U_b - U_a$

$= -W^F$ independent of path

$* \quad = +W^{Ext}$

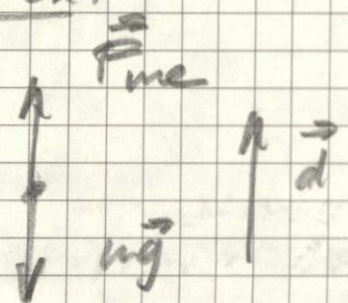


$\Rightarrow \sum \vec{F} = \vec{0}$

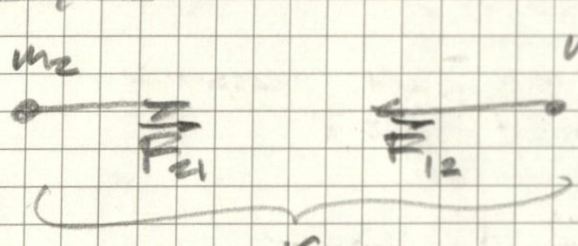
$\Rightarrow$ no acceleration

$\rightarrow$ moving at constant velocity

Chalk Ex.



Gravity Ex.



$\vec{F}_{12} = -\vec{F}_{21}$, $F = \frac{Gm_1m_2}{r^2}$, $U(r) = \frac{-Gm_1m_2}{r}$

U increases as r decreases ...

Ex. Potential Energy of two charged objects

q_1 $\qquad \vec{d} \qquad$ q_2 same signs

q_1 & q_2 opposite ⇒ reversed

U increases as r decreases

Potential Energy of Charges

$U(r) = \frac{q q_0}{4\pi \varepsilon_0 r}$ no more absolute value

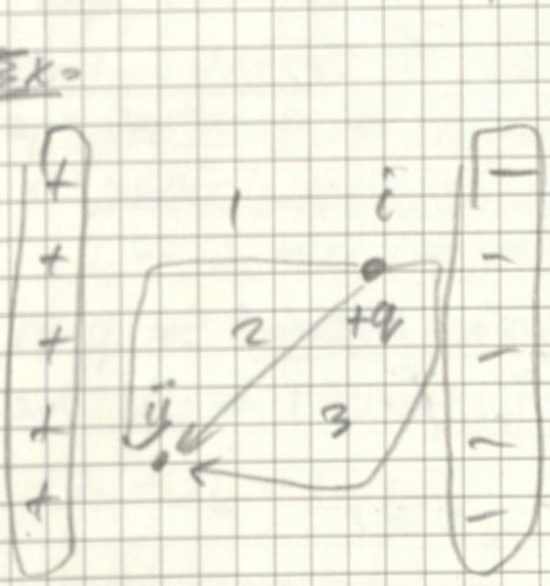
works w/ signs!

$\Delta U = mg\Delta y$

$U(y) = mgy$ ambiguous on its own

must make a choice in coordinates before making this equality

Ex.



which path requires most work?

All the same!

independent of path

Reading Notes:

23.2: Electric Potential

Potential (V): potential energy per unit charge

* closely related to $\vec{E}$ *

$$V = \frac{U}{q_0} \quad \text{or} \quad U = q_0 V$$

scalar values

SI. volt (V) = 1 J/C

Form:

$$\frac{W_{a\to b}}{q_0} = -\frac{\Delta U}{q_0} = -\left(\frac{U_b}{q_0} - \frac{U_a}{q_0}\right) = -(V_b - V_a) = V_a - V_b$$

$V_{ab} = V_a - V_b$: potential of a w/respect to b

"work (J) done by electric force when a unit charge (1C) moves from a→b

"work (J) that must be done to move a unit charge (1C) slowly from b→a against the electric force"

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}, \quad V = \frac{1}{4\pi\varepsilon_0}\sum_i \frac{q_i}{r_i}$$

one point charge — collection of point charges

$$V = \frac{1}{4\pi\varepsilon_0}\int \frac{dq}{r}$$

continuous distribution of charge

Finding Electric Potential from Electric Field:

$$W_{a\to b} = \int_a^b \vec{F}\cdot d\vec{l} = \int_a^b q_0 \vec{E}\cdot d\vec{l}$$

$$V_{ab} = V_a - V_b = \int_a^b \vec{E}\cdot d\vec{l} = \int_a^b E\cos\phi \, dl$$

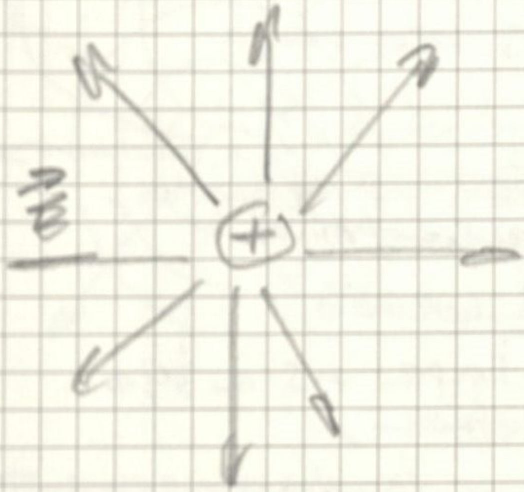
Electric Potential difference (conservative)

angle btw $\vec{E}$ and $d\vec{l}$

Note: $V_a - V_b = -\int_b^a \vec{E}\cdot d\vec{l}$

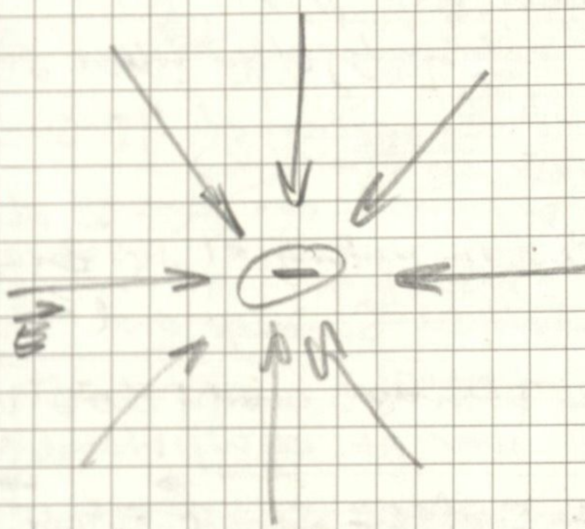
Interpretation:

- If $\int_a^b \vec{E}\cdot d\vec{l} > 0$, $\vec{E}$ does $W > 0$ on q from a to b, resulting in decreasing U and therefore decreasing V
- hence $V_b < V_a$ and $V_a - V_b$ is positive
- vice versa



V increases as you move inward

V decreases as you move outward



V decreases as you move inward

V increases as you move outward

Electron Volts:

magnitude e of electron charge can be used to define a useful unit of energy (NOT potential)

$$U_a - U_b = q(V_a - V_b) = qV_{ab}$$

if $q = e = 1.602\cdot10^{-19}\,C$

and $V_{ab} = 1\,V = 1\,J/C$

$$U_a - U_b = (1.602\cdot10^{-19}\,C)(1\,J/C) = 1.602\cdot10^{-19}\,J$$

which is an electron volt (1 eV)

$$1\,eV = 1.602\cdot10^{-19}\,J$$

meV, keV, MeV, GeV, and TeV often used

Reading Notes: 9.25.24

23.4: Equipotential Surfaces

Electric Field Lines → help visualize Electric fields

Equipotential Surfaces → help visualize electric Potential

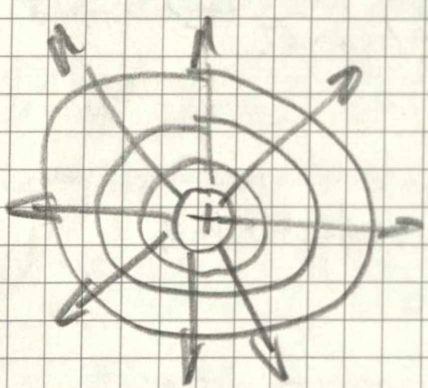
- like a topographic map
- contour plot
  - close together when steep vice versa
- cross-section of 4D-surface x-y-z-V (for constant V)
- no point can have 2 different potentials, so equipotential surfaces never touch

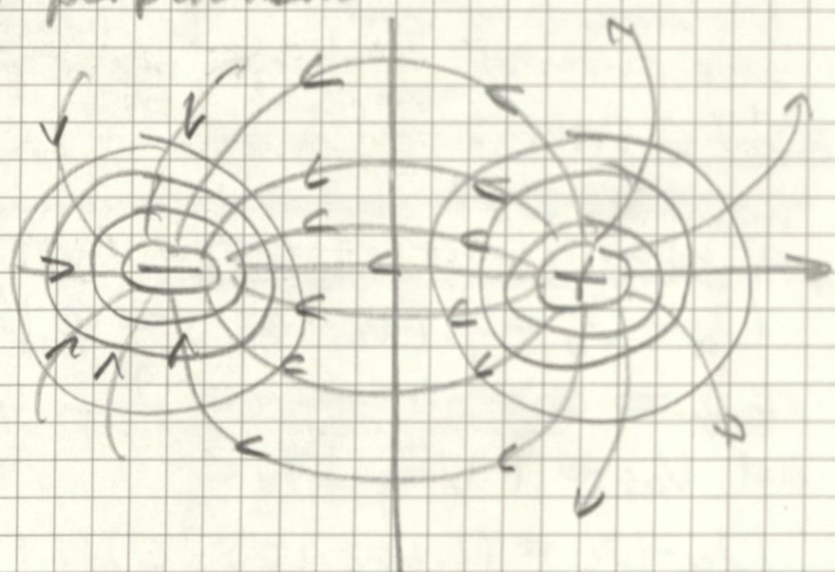
Equipotential Surfaces and Field Lines:

Since potential energy does not change as a q_0 moves over an equipotential surface,

$\vec{E} \perp$ surface so $\vec{F} = q_0\vec{E} \perp q_0$'s displacement

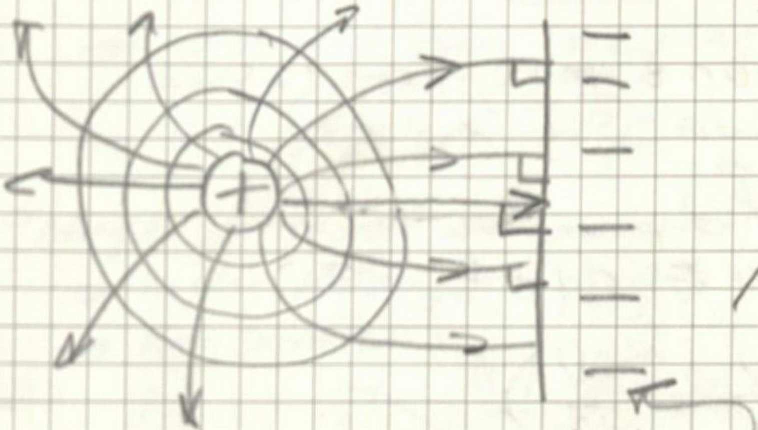
∴ Field lines and equipotential surfaces are always mutually perpendicular

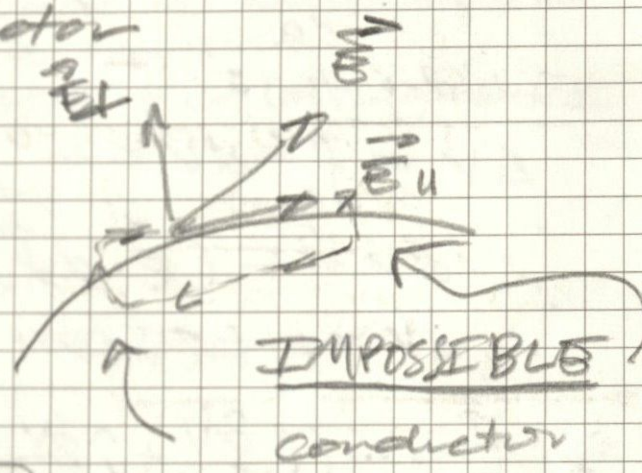




Equipotentials and Conductors:

- When all charges are at rest, the surface of a conductor is always an equipotential surface
- When all charges are at rest, $\vec{E}$ just outside a conductor must be $\perp$ to the conductor at every point
- $\vec{E} = \vec{0}$ inside the conductor





conductor

- When all charges are at rest, entire solid volume of conductor is at the same potential
  - equipotential volume

Theorem: If a conductor contains a cavity and no charge is present inside the cavity, there is no net charge anywhere on the surface of the cavity

- proof by contradiction w/ Gauss's Law
- $\vec{E}$ must be $\vec{0}$ inside cavity
- σ must be 0 on the wall of the cavity

23.5 : Potential Gradient

$\vec{E}$ and V are closely related:

$$\Delta V = -\int_a^b \vec{E}\cdot d\vec{l}$$

$\vec{F}$ and U are closely related:

$$\Delta U = -\int_a^b \vec{F}\cdot d\vec{l}$$

Extending this:

$$\Delta V = \int_a^b dV = -\int_a^b \vec{E}\cdot d\vec{l}$$

$$\therefore dV = -(\vec{E}\cdot d\vec{l})$$

$$\therefore dV = -(E_x dx + E_y dy + E_z dz)$$

$$\Rightarrow \vec{E} = -\left(\hat{i}\frac{\partial V}{\partial x} + \hat{j}\frac{\partial V}{\partial y} + \hat{k}\frac{\partial V}{\partial z}\right)$$

$$\left(\vec{\nabla} f = \left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial x} + \hat{k}\frac{\partial}{\partial z}\right) f\right)$$

$$\Rightarrow \vec{E} = -\vec{\nabla} V$$

so is $\vec{F} = -\vec{\nabla} U$?

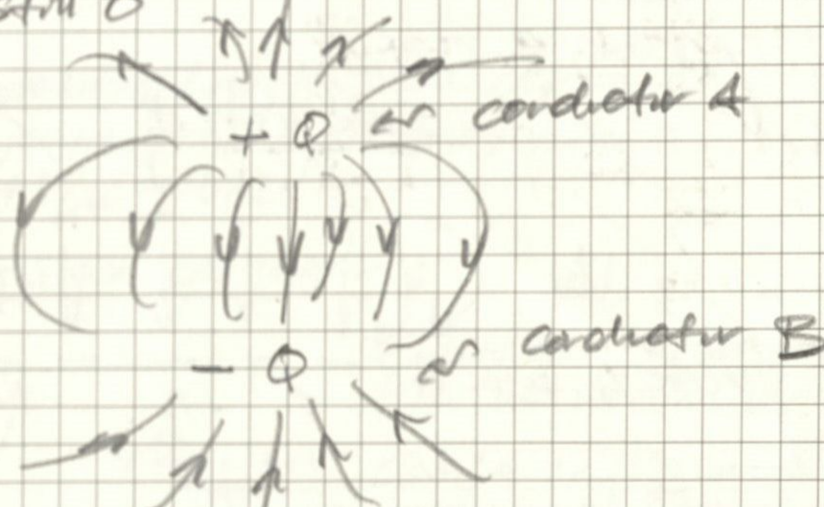
Reading Notes: 9.25.24

24.1: Capacitors and Capacitance

Capacitor: any 2 conductors separated by an insulator or vacuum.

→ each capacitor initially has zero net charge
→ electrons are transferred from one conductor to the other to charge the capacitor
→ conductors then have charges of equal magnitude and opposite sign
→ net charge remains 0

"charge Q stored on capacitor"
→ higher potential has +Q and lower −Q
→ net still 0



* charge by connecting the 2 wires to opposite terminals of a battery

V_{ab} = voltage of the battery
↳ of the conductor

Capacitance: ratio of charge to potential difference

$$C = \frac{Q}{V_{ab}} \qquad \text{SI: farad } 1F = \frac{1C}{1V} = \frac{C}{V}$$

* don't confuse w/ coulombs

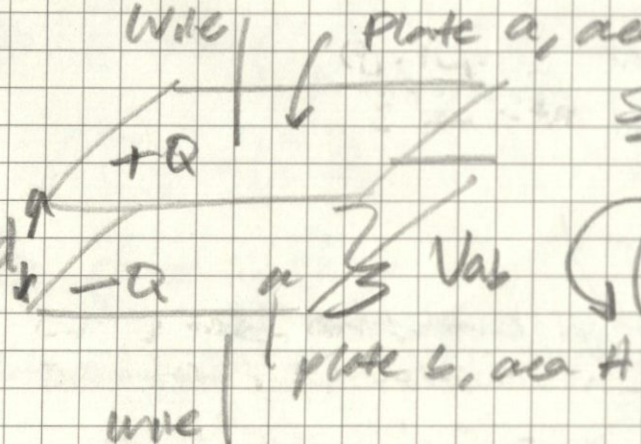
* greater the C, greater the stored energy *

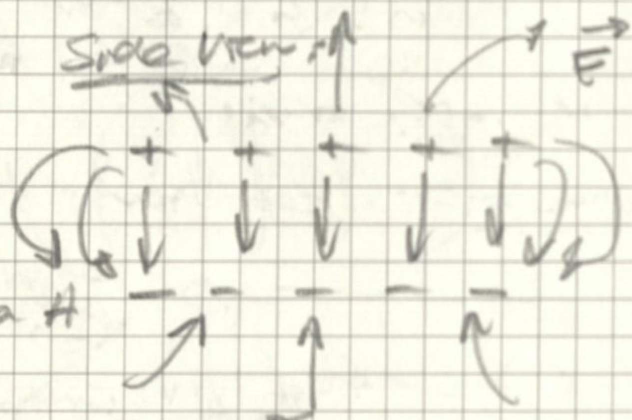
→ measure of the ability of a capacitor to store energy

Calculating Capacitance: Capacitors in a Vacuum:

empty space separates capacitors

Parallel Plate Capacitor: Simplest form





Superposition of $\vec{E}$ + Gauss: $E = \frac{\sigma}{\epsilon_0}$

where $\sigma = \frac{Q}{A}$

So:

$$E = \frac{\sigma}{\epsilon_0} = \frac{Q}{\epsilon_0 A}$$

$$V_{ab} = Ed = \frac{Qd}{\epsilon_0 A}$$

$$C = \frac{Q}{V_{ab}} = \frac{A\epsilon_0}{d}$$

Note: $\epsilon_0 = 8.85 \cdot 10^{-12} \frac{F}{m}$

$$F = \frac{C^2}{N \cdot m} = \frac{C^2}{J}$$

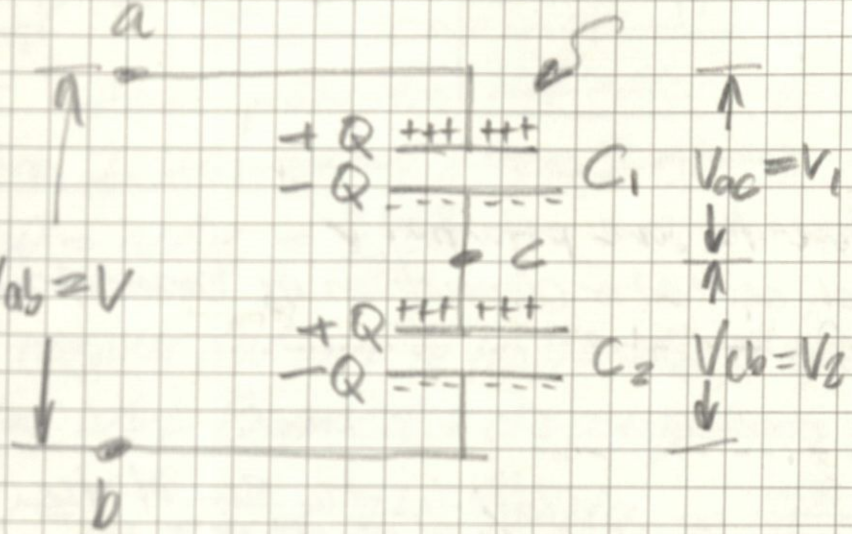
Reading Notes:

9.26.24

24.2. Capacitors in Series and Parallel

Capacitors in Series:

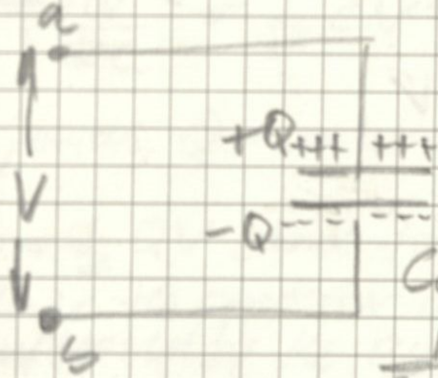
series connection



- The capacitors have the same charge Q
- Their potential differences add:

$$V_{ac} + V_{cb} = V_{ab}$$

equivalent single capacitor:



Equivalent capacitance is less than individual capacitances

Charge is the same for individual capacitors

$$C_{eq} = \frac{Q}{V}$$

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$$

Potential Differences:

$$V_{ac} = V_1 = \frac{Q}{C_1}, \quad V_{cb} = V_2 = \frac{Q}{C_2}$$

$$V_{ab} = V = V_1 + V_2 = Q\left(\frac{1}{C_1} + \frac{1}{C_2}\right)$$

$$\rightarrow \frac{V}{Q} = \frac{1}{C_1} + \frac{1}{C_2}$$

Capacitors in Series:

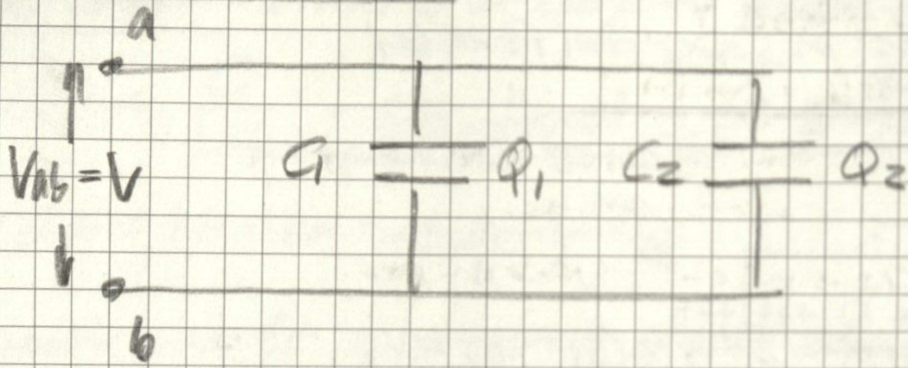
capacitance of individual capacitors

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \dots$$

Note: $C_{eq} < C_n$

$$V_{tot} = V_1 + V_2 + V_3 + \dots$$

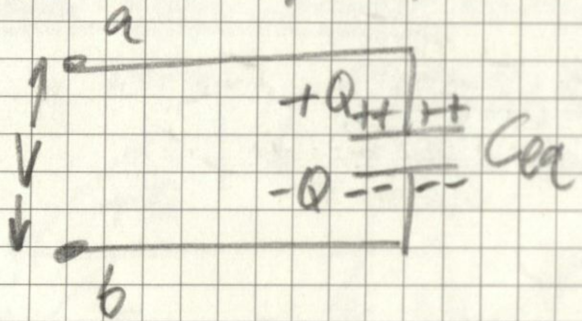
Capacitors in Parallel:



- The capacitors have the same potential V
- The charge on each capacitor depends on its capacitance:

$Q_1 = C_1 V$, $Q_2 = C_2 V$

equivalent single capacitor:



Charge is the sum of the individual charges:

$$Q = Q_1 + Q_2$$

Equivalent capacitance:

$$C_{eq} = C_1 + C_2$$

Total Charge:

$$Q = Q_1 + Q_2 = (C_1 + C_2) V$$

$$\Rightarrow \frac{Q}{V} = C_1 + C_2$$

Capacitors in Parallel:

$$C_{eq} = C_1 + C_2 + C_3 + \dots$$

equivalent capacitance of parallel combination

capacitance of individual capacitors

Note. $C_{eq} > C_n$

$$Q_{total} = Q_1 + Q_2 + Q_3 + \dots$$

24.3. Energy Storage in Capacitors and Electric-Field Energy

Capacitors: work by storing U

$V = \frac{Q}{C}$ ← final charge after charging; C ← capacitance; V ← final potential diff after charging

At an Intermediary Stage: $Q = q$, $V = v$

$dW = v\,dq = \frac{q\,dq}{C}$

requires dW work to transfer dq charge

Total Work Required: charge q from 0 to Q

$$W = \int_0^W dW = \frac{1}{C}\int_0^Q q\,dq = \frac{Q^2}{2C}$$

work to charge a capacitor

Define U of uncharged capacitor as 0: $W = U$

$$U = \frac{Q^2}{2C} = \frac{1}{2}CV^2 = \frac{1}{2}QV$$

U - joules, Q - coulombs, C - farads ($\frac{C}{V}$), V - volts

Note. $U = \frac{1}{2}QV$ shows that total W required to charge the capacitor is equal to the total charge Q multiplied by the average potential diff $\frac{1}{2}V$ during the process

Capacitor		Spring
$\frac{1}{C}$	~	k
$U = \frac{1}{2}\left(\frac{Q^2}{C}\right)$	~	$\frac{1}{2}kx^2$
Q	~	x
charging process	~	stretching

Hi ARPA I ♡ U
-Toni

Applications of Capacitors: Energy Storage

- energy is stored in V
- energy is released by connecting the capacitors

ex. flash photography ↳ by a conducting path

Electric-Field Energy

- the energy is stored in the field b/w the plates

energy density: energy per unit volume

$U = \frac{1}{2}CV^2$, plate area $= A$, distance $= d$

$$u = \frac{\frac{1}{2}CV^2}{Ad}$$

↳ energy density

Recall. $C = \frac{\varepsilon_0 A}{d}$, $V = Ed$

$$\Rightarrow u = \frac{1}{2}\varepsilon_0 E^2$$

↳ magnitude of electric field

* valid for any electric-field configuration in vacuum
  → we have only discussed parallel-plate capacitors

CAUTION. Electric field energy is just a different way of thinking about electric potential energy
→ the same thing measured

Reading Notes: 9.29.24

24.4. Dielectrics

dielectric: nonconducting material b/w conducting plates of a conductor

adding a dielectric decreases ΔV, thus increasing C

original capacitance w/ dielectric:

$$C_0 = \frac{Q}{V_0} \qquad C = \frac{Q}{V}$$

Q is the same and $V < V_0$, so $C > C_0$

Dielectric Constant: When Q is constant:

$$K = \frac{C}{C_0} \qquad Q = C_0V_0 = CV, \quad \frac{C}{C_0} = \frac{V_0}{V}$$

$$\Rightarrow V = \frac{V_0}{K}$$

w/ dielectric ΔV reduced by a factor of K
↳ potential difference

K = pure number:
→ K=1 in vacuum by def
→ greater K → worse dielectric

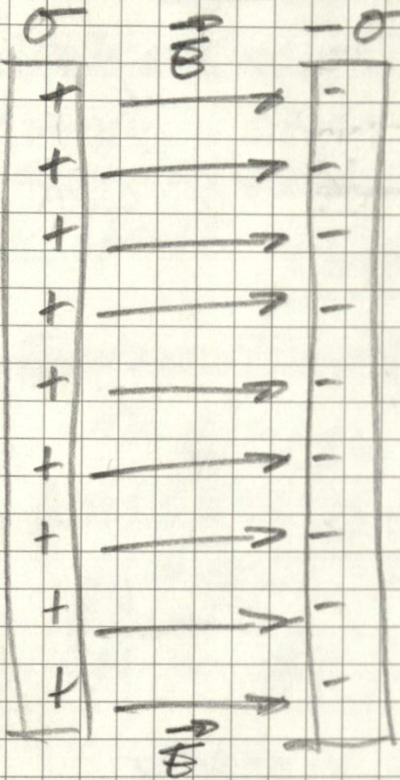
Induced Charge and Polarization:

$$E = \frac{E_0}{K} \quad \text{(when Q is constant)}$$

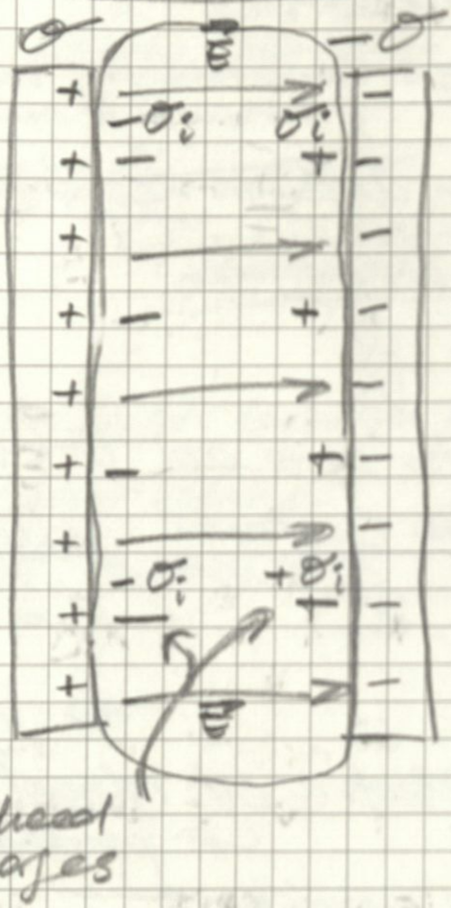
Since V decreases by a factor of K, so does E

Polarization: since $E < E_0$, σ must decrease. the actual charges cannot decrease so instead they redistribute

Vacuum:



Dielectric:



Relationships:

$$E_0 = \frac{\sigma}{\varepsilon_0}, \quad E = \frac{\sigma - \sigma_i}{\varepsilon_0}$$

$$\Rightarrow \sigma_i = \sigma\left(1 - \frac{1}{K}\right)$$

$\varepsilon = K\varepsilon_0$ (permittivity of the dielectric)

$\Rightarrow E = \frac{\sigma}{\varepsilon}$ (within the dielectric)

So:

$$C = KC_0 = K\varepsilon_0 \frac{A}{d} = \varepsilon \frac{A}{d}$$

capacitance w/ dielectric

capacitance in vacuum

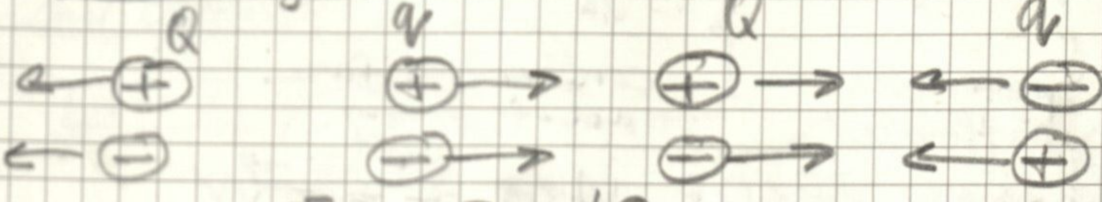
$$u = \frac{1}{2} K\varepsilon_0 E^2 = \frac{1}{2}\varepsilon E^2$$

electric energy density in dielectric

Exam 1 Review

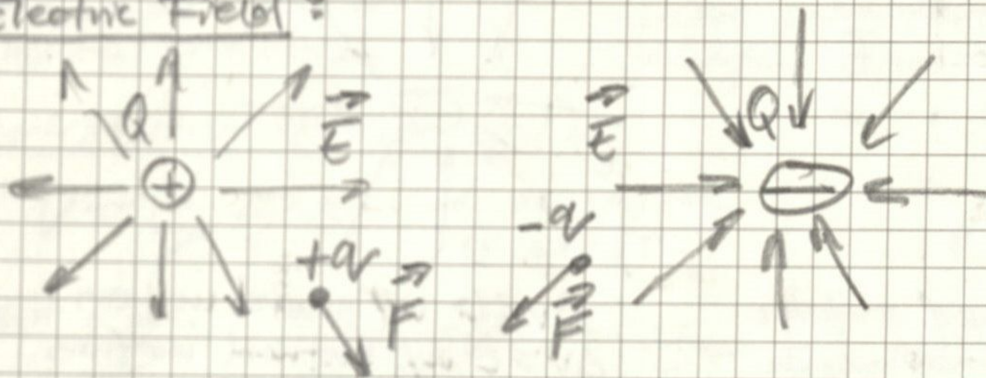
9.30.24

Point Charges:



$\vec{F} = q\vec{E}$, $\vec{E} = \frac{kQ}{r^2}\hat{r}$

Electric Field:



$q > 0$, $\vec{F}$ and $\vec{E}$ in same direction
$q < 0$, $\vec{F}$ and $\vec{E}$ in opposite directions

Gauss's Law: application of div. thm

$$\oiint_{\partial V} \vec{E}\cdot d\vec{S} = \frac{1}{\varepsilon_0}\iiint_V \rho \, dV$$

ρ ↳ volume charge density C/m3

In this course:

$$\sum \Phi = \oint \vec{E}\cdot\hat{n}\, dA = \frac{1}{\varepsilon_0} Q_{encl}$$

Most of the time:

$$EA = \frac{Q}{\varepsilon_0}$$

Maybe

$$EA\cos\phi = \frac{Q}{\varepsilon_0}$$

if $\vec{E}$ and $\hat{n}$ in different directions

Cases - spherical, cylindrical, planar (all use symmetry)

Electric Potential:

$$V = \frac{KQ}{r}, \quad \Delta V = V_b - V_a = -\int_a^b \vec{E} \cdot d\vec{l} \quad \left(V = \frac{J}{C}\right)$$

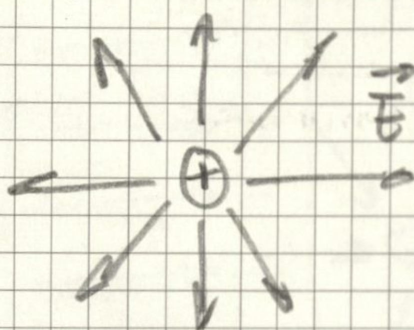
point charge

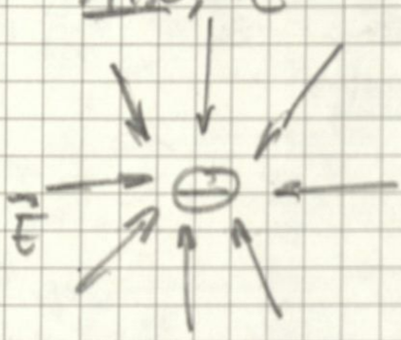
"potential difference"

conservative force
ind of path

Charges move towards $\Delta V < 0$:

Also, $\vec{E} = -\vec{\nabla} V$





$\Delta V > 0$ w/ inward motion
$\Delta V < 0$ w/ outward motion

$\Delta V < 0$ w/ inward motion
$\Delta V > 0$ w/ outward motion

Potential Energy:

$$U = qV \Leftrightarrow V = \frac{U}{q} \Leftrightarrow \Delta U = q\Delta V$$

$$W = \int \vec{F} \cdot d\vec{l} = \int q\vec{E} \cdot d\vec{l} = -\Delta U \quad (J = Nm)$$

Conductors: (in electrostatic equilibrium)

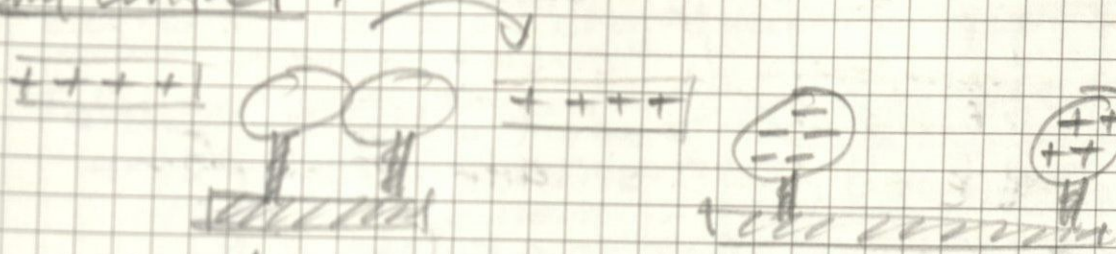
1. Free movement of e^- → easily transfer charges
2. $\vec{E}$ inside is $\vec{0}$ → internal $\vec{E}$ causes charges to redistribute until field cancels
3. All excess charge reside on surface (σ)
4. All points on surface and inside conductor have the same V ($\Delta V = 0$)
5. Shields interior from $\vec{E}$ → internal $\vec{E} = \vec{0}$

Insulators:

1. e^- cannot move → no movement of charges
2. Can become polarized (have separation of pos/neg charges) which cause $\vec{E}$ inside the insulator
   → useful in dielectrics

Practice Exam 1.　　　　9.30.24

Charges can be induced (moved) w/o any contact:



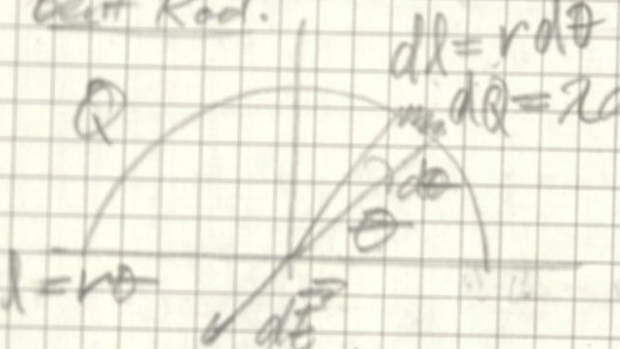
rod induces charges on neutral conductors

when separated charge remains

Superposition of Potentials:

$\sum V = \frac{k}{r}\sum Q_i$　if multiple Q exist

Bent Rod.



$E = \frac{kQ}{r^2}$, $dE = \frac{k\,dQ}{r^2}$

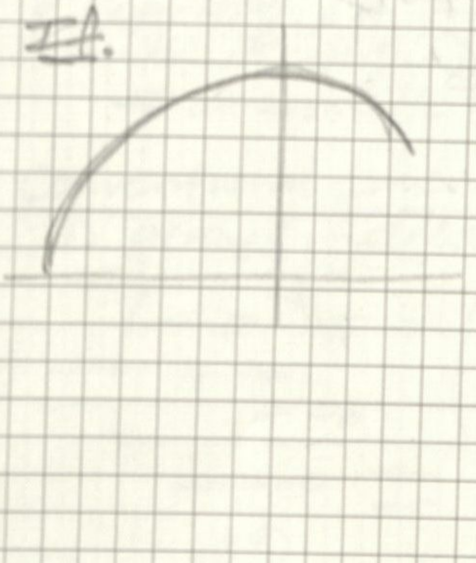
$\lambda = \frac{Q}{l} = \frac{Q}{r\theta}$, $Q = \lambda r\theta$

$\Rightarrow dE = \frac{k\lambda r\,d\theta}{r^2} = \frac{k\lambda\,d\theta}{r}$

$d\vec{E}_x = -\frac{k\lambda\,d\theta}{r}\cos\theta\,\hat{i}$, $d\vec{E}_y = -\frac{k\lambda\,d\theta}{r}\sin\theta\,\hat{j}$

$E_x = 0$ by symmetry, $E_y = \int_0^{\pi} -\frac{k\lambda\,d\theta}{r}\sin\theta = -\frac{k\lambda}{r}\int_0^{\pi}\sin\theta\,d\theta$

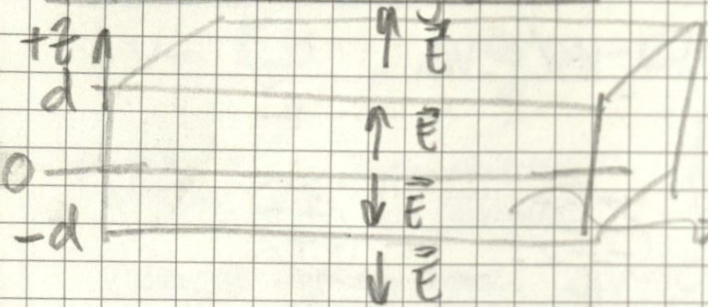
If.



$E_y = \int_{\frac{\pi}{4}}^{\pi} dE_y$

$E_x = \int_{\frac{\pi}{4}}^{\pi} dE_x$

Infinite Insulating Sheet:



b/c of varying $\vec{E}$ inside sheet

→ more E on top than below so vector points down

* Use Cylindrical Gaussian Surfaces to calculate $\vec{E}$ *

→ endcaps are important

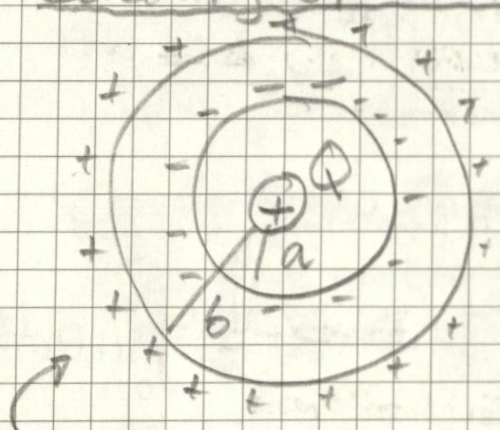
→ side is $\perp$ to $\hat{n}$ so $\Phi = 0$ ∴ can ignore

Use $\Delta V = -\int_a^b \vec{E} \cdot d\vec{l}$ to calculate V

if V is 0 initially and unknown at a point z:

$$0 - V(z) = \int_0^z E\, dz$$

Conducting Spherical Shell:



induces charge

$\underline{r<a}$: $Q_{encl} = Q$

$\underline{r=a}$: $V = \frac{kQ}{b}$ ← $\Delta V = 0$ inside conductor $(\vec{E} = \vec{0})$

$\underline{a<r<b}$: $Q_{encl} = 0$

$$V = \frac{kQ}{b}$$

$\underline{r>b}$: $Q_{encl} = Q$

$$V = \frac{kQ}{r}$$

$\underline{0<r<a}$:

$$\Delta V = V_f - V_i = -\int_{r_i}^{r_f} E\, dr = -$$

Reading Notes: 10.2.24

25.1. Current

current: motion of charges from one region to another

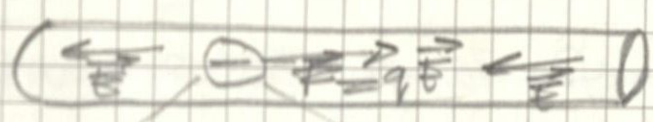
for now focus on currents in conductors

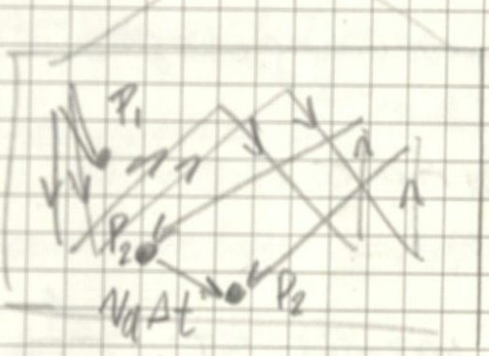
→ in electrostatic situations, $\vec{E}=\vec{0}$ and there is no current in conductors

→ does not mean no motion inside conductor — e^-'s move erratically like gas particles

→ results in no net motion and thus no current

Suppose: $\vec{E}$ established inside conductor





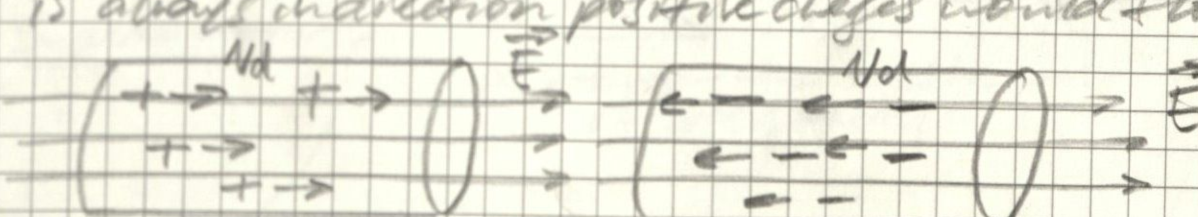
$\vec{v}_d$: drift velocity

→ net motion difference b/w e^- random movement with and without $\vec{E}$

Direction of Current:

- $\vec{E}$ does work on moving charges causing a $\vec{v}_d$.
- Resulting KE transfered to material by collisions with its ions — material heats up
  - → unavoidable byproduct of current flow

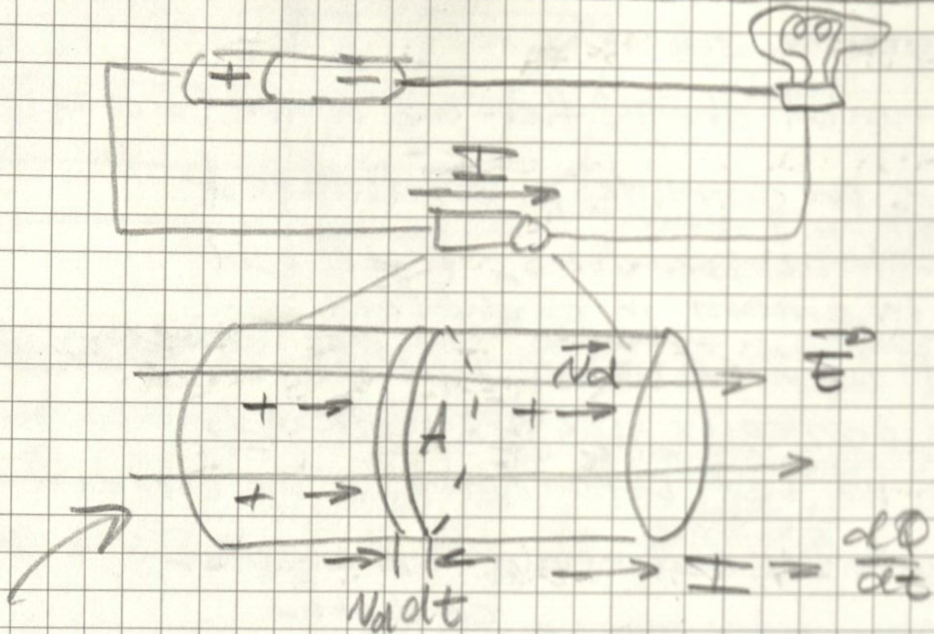
Conventional current: convention of current flow is always in direction positive charges would flow



$I = \frac{dQ}{dt}$ (definition of current)

SI: Ampere ($\frac{C}{s}$) CAUTION: not a vector! - no single vector could describe circuit motion

Current, Drift Velocity, Current Density:



* time rate of charge transfer through cross-sectional area $\underline{A}$

$\underline{n}$: concentration of charges — n moving charges per unit volume. $\underline{\text{SI}}$: m^{-3}

$$dQ = q(nAv_d\,dt) = nqv_dA\,dt$$

$$\Rightarrow I = \frac{dQ}{dt} = nqv_dA$$

Current Density: $J = \frac{I}{A} = nqv_d$ $\underline{\text{SI}}$. $\frac{A}{m^2}$

Current:

$$I = \frac{dQ}{dt} = n|q|v_dA$$ scalar!

Current Density:

$$\vec{J} = nq\vec{v}_d$$ vector!

25.2 Resistivity

Ohm's Law: idealized model relating $\vec{E}$ as directly proportional to $\vec{J}$

* assumed valid in this class

$$\rho = \frac{E}{J}$$

magnitude of electric field

magnitude of current density caused by electric field

resistivity

* greater the ρ, greater the E required to cause J

SI: $V \cdot \frac{m}{A} = \Omega \cdot m$ "ohm meters"

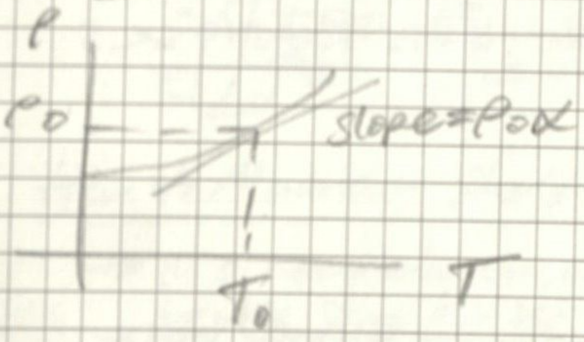
Resistivity and Temperature

$$\rho(T) = \rho_0[1 + \alpha(T - T_0)]$$

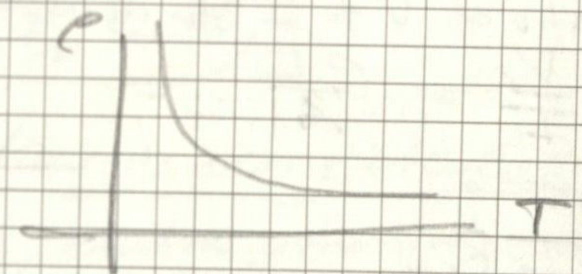
temperature dependence of resistivity

* need more here if applicable

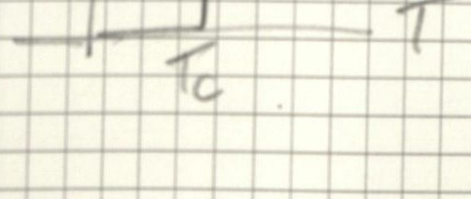
Metal:



Semiconductor:



Superconductor:

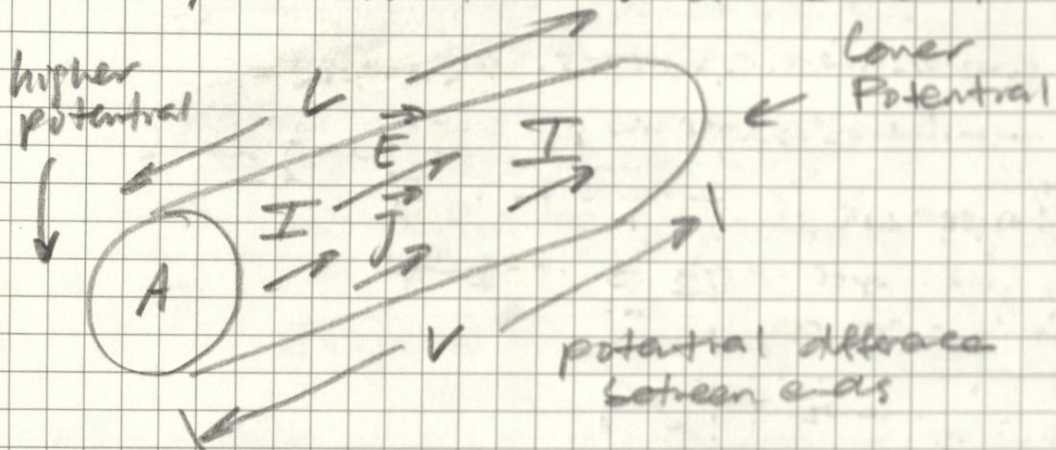


α = temperature coefficient of resistivity

25.3: Resistance

$\vec{E} = \rho \vec{J}$ when Ohm's Law is obeyed

* however, much more interested in total current I and potential difference V b/w ends of conductor



* current flows from higher to lower potential

If J and E are uniform throughout conductor:

$I = JA$ and $V = EL$

<u>So</u>:

$$\frac{V}{L} = \frac{\rho I}{A} \text{ or } V = \frac{\rho L}{A} I$$

when ρ is constant I proportional to V

* This ratio of V to I for a conductor is its <u>Resistance</u>:

$$R = \frac{V}{I} = \frac{\rho L}{A}$$

<u>Ohm's Law</u>:

$$V = IR$$

<u>Interpreting Resistance</u>: fluid flow — narrow hose vs. fat hose

Reading Notes: 10.3.24

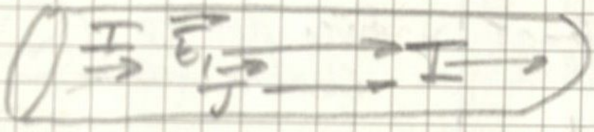
25.4: Electromotive Force and Circuits

Complete Circuit: conductor in part of a path that forms a closed loop

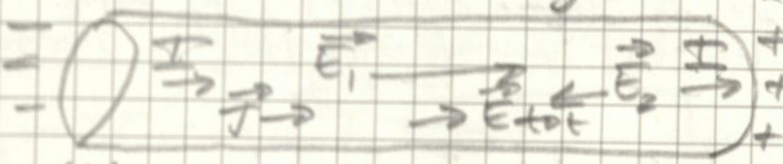
* For a conductor to have a steady current it must be a part of a complete circuit *

→ else it always tries to return to its electrostatic equilibrium w/ $\vec{E}_{inside} = \vec{0}$ and all charges on surface

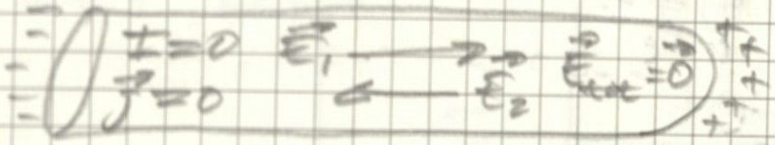
a) an $\vec{E}_1$ produced inside an isolated conductor causes an I



b) The current causes charges to build up on either ends producing an $\vec{E}_2$, reducing I



c) After a short time $E_2 = E_1$, $\vec{E}_{tot} = \vec{0}$ and the current I stops



How to Maintain Steady Flow in a complete circuit?

→ in a complete circuit, electric potential energy must be the same at the start and after one full revolution

→ there also must be a constant decrease in potential energy throughout the conducting material

→ must have a part of the circuit where the potential energy increases

- like a water pump in a water fountain

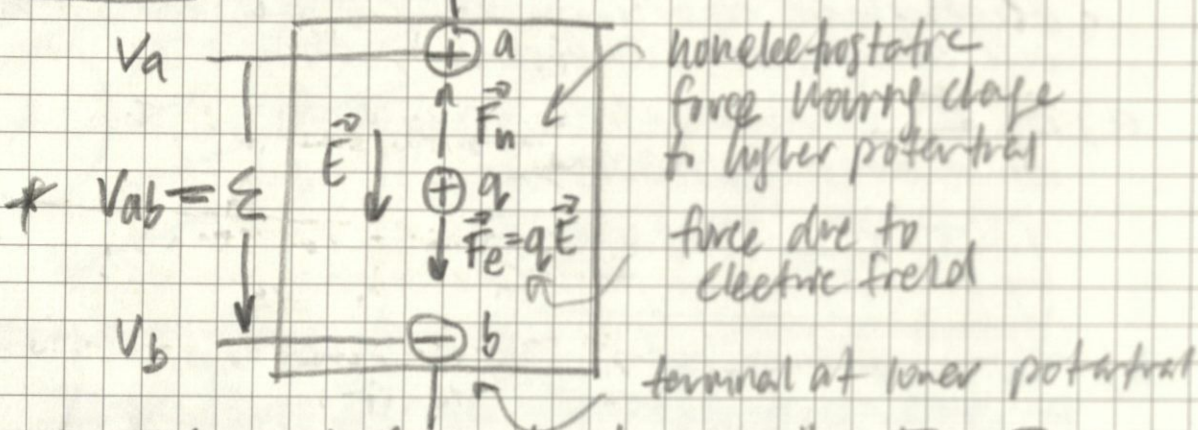
Electromotive Force = emf ($\mathcal{E}$)

- "force" that makes current flow from lower to higher potential
  - ⇒ like water pump

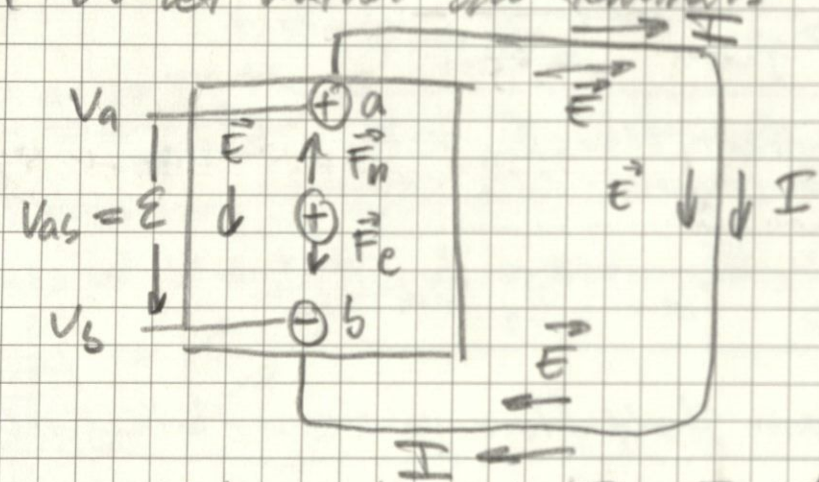
$SI: V = \frac{J}{C}$; not a force, same units as potential

ex. batteries, generators, solar cells, ...

Ideal emf:



* when not apart of a closed circuit, $F_n = F_e$ and no net motion b/w terminals



* when apart of a real circuit $F_n > F_e$ and does work on charges

$$\mathcal{E} = V_{ab} = IR$$

Internal Resistance = resistance of non-ideal emf

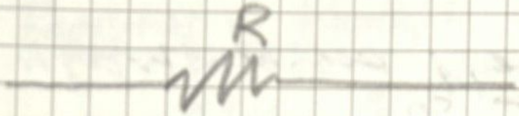
$$V_{ab} = \mathcal{E} - Ir$$

in a non-ideal emf

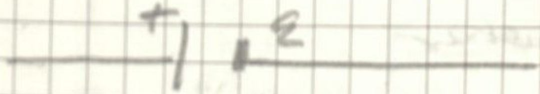
$$\mathcal{E} - Ir = IR \Rightarrow I = \frac{\mathcal{E}}{R + r}$$

Circuit Diagram Symbols:

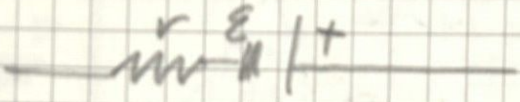
Conductor with negligible resistance



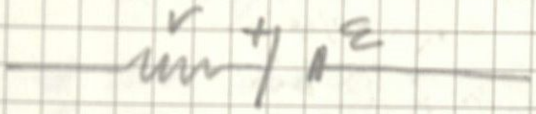
Resistor



Source of emf (+ indicates terminal w/ higher potential)



Source of emf with internal resistor r



can be placed on either side

Voltometer (measures potential difference b/w terminals)

Ammeter (measures current through it)

Potential Change Around a Circuit:

$$\varepsilon - Ir - IR = 0$$

* the net potential change around a circuit must be 0

25.5. Energy and Power in Electric Circuits

$dQ = I\,dt$ charge passes through a circuit so:

$V_{ab}\,dQ = V_{ab} I\,dt$ potential energy passes through a circuit so:

Power: time rate of energy transfer

$P = V_{ab} I$ SI: Watt $W = \frac{J}{s}$

Power Input to Pure Resistance

voltage across resistor

$P = V_{ab} I = I^2 R = \frac{V_{ab}^2}{R}$

power delivered to a resistor

current in resistor

resistance of resistor

Power Output of a Source:

$P = V_{ab} I$, $V_{ab} = \mathcal{E} - Ir$

$P = V_{ab} I = \mathcal{E} I - I^2 r$

Power Input to a Source:

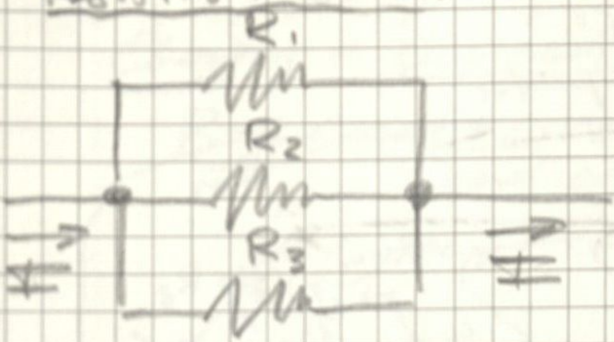
$P = V_{ab} I$, $V_{ab} = \mathcal{E} + Ir$

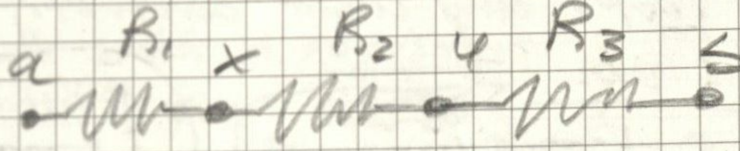
$P = V_{ab} I = \mathcal{E} I + I^2 r$

Reading Notes: 10.6.24

26.1. Resistors in Series and in Parallel

Resistors in Parallel: Resistors in Series:





* In series with parallel combination
* In parallel with series combination

Equivalent Resistance: a single resistor that could replace the entire combination

$$V_{ab} = I R_{eq} \Rightarrow R_{eq} = \frac{V_{ab}}{I}$$

Resistors in Series:

$$V_{ax} = IR_1, \quad V_{xy} = IR_2, \quad V_{yb} = IR_3$$

$$\Rightarrow V_{ab} = V_{ax} + V_{xy} + V_{yb} = I(R_1 + R_2 + R_3)$$

$$\Rightarrow R_{eq} = \frac{V_{ab}}{I} = R_1 + R_2 + R_3$$

$$R_{eq} = R_1 + R_2 + R_3 + \dots, \quad R_{eq} > R_n$$

Resistors in Parallel:

$$I_1 = \frac{V_{ab}}{R_1}, \quad I_2 = \frac{V_{ab}}{R_2}, \quad I_3 = \frac{V_{ab}}{R_3}$$

$$\Rightarrow I = I_1 + I_2 + I_3 = V_{ab}\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)$$

$$\rightarrow \frac{1}{R_{eq}} = \frac{I}{V_{ab}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

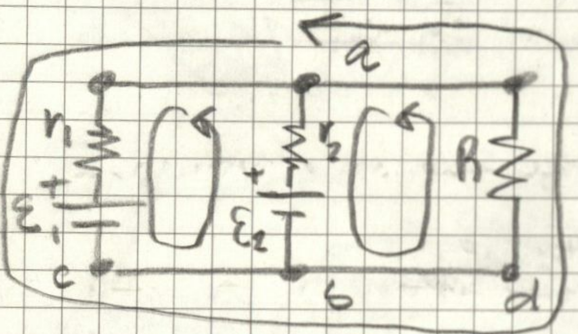
$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \dots, \quad R_{eq} < R_n$$

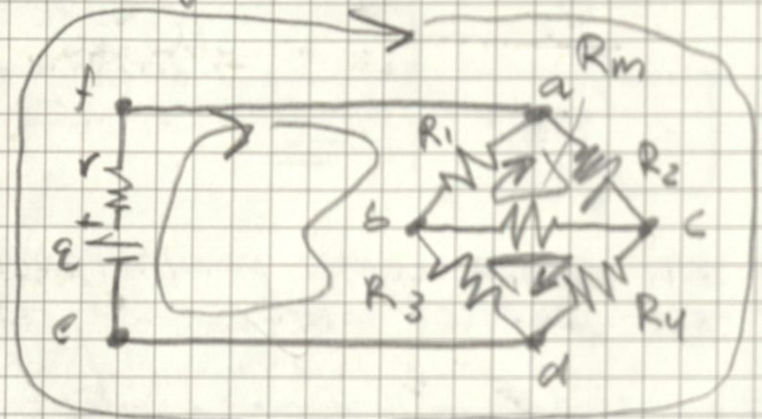
Reading Notes: 10.8.24

26.2 Kirchoff's Rules

junction: a point in a circuit where 3 or more conductors meet

loop: any closed conducting path





- 3 loops
- 2 junctions

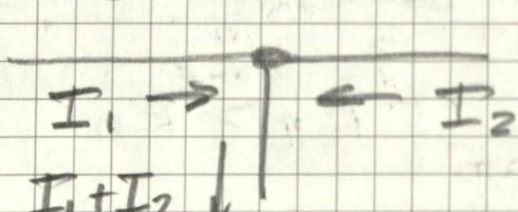
- 4 loops
- 4 junctions

Junction Rule: $\Sigma I = 0$ & conservation of energy

Loop Rule: $\Sigma V = 0$ & conservation of charge

↳ including emfs and resistors

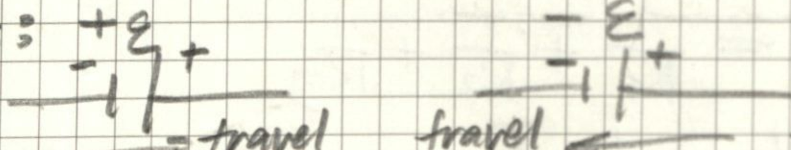
Direction for Junction Rule:



Sign Conventions for Loop Rule:

- assume a direction for current in each branch of circuit
  → mark it on diagram of circuit

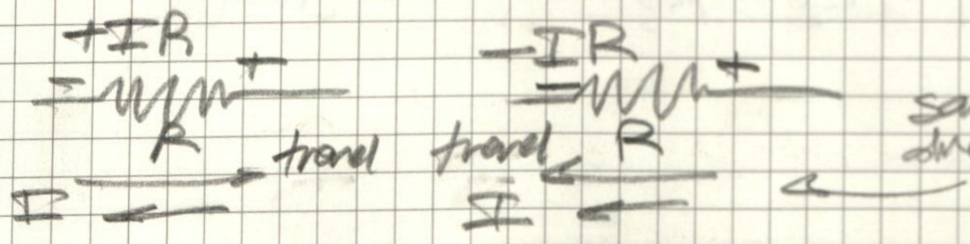
emfs:



* travel is direction we imagine going around circuit
→ not always I

opposite direction

↳ I



same direction

Reading Notes: 10.10.24

26.4: R-C Circuits

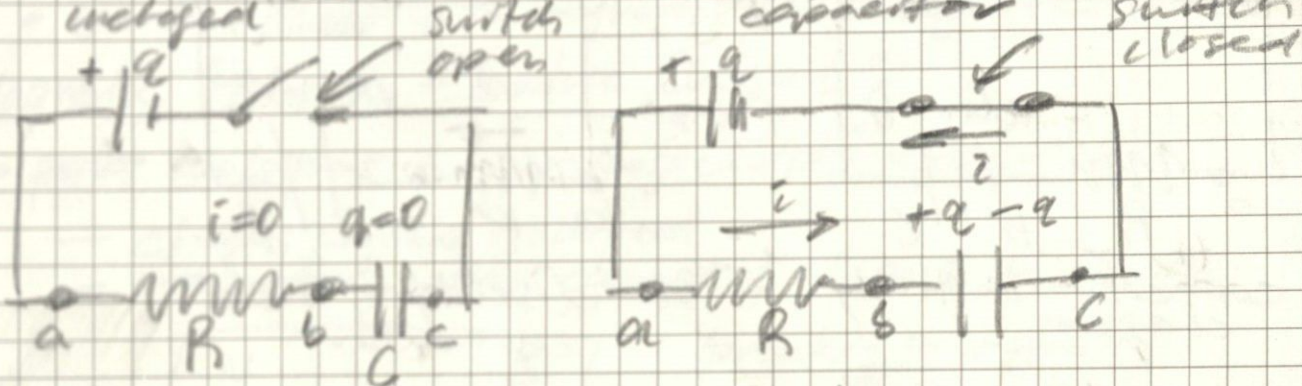
* emf's and resistors do change w/ time

Charging a Capacitor:

R-C Circuit: circuit with a resistor and capacitor in series

→ battery - constant emf and $r = 0$ internal resistance

a) Capacitor initially uncharged b) charging the capacitor



CAUTION: lowercase letters means time-varying

* Long calculus derivation in book

$$q = C\mathcal{E}(1 - e^{-t/RC}) = Q_f(1 - e^{-t/RC})$$

R-C circuit charging capacitor

$$i = \frac{dq}{dt} = \frac{\mathcal{E}}{R} e^{-t/RC} = I_0 e^{-t/RC}$$

Note:

t: time since switch closed

$Q_f = C\mathcal{E}$

I_0: initial current ; $I_0 = \frac{\mathcal{E}}{R}$

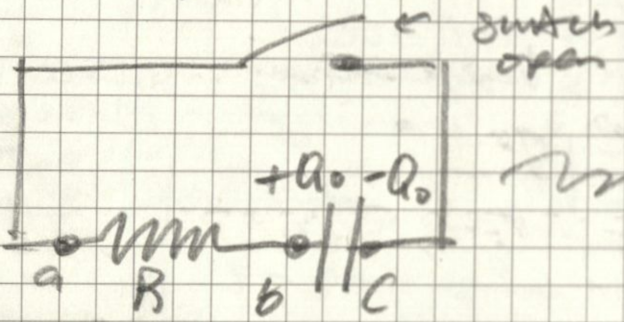
* i exponentially decreases as the capacitor charges *

Time Constant: after a $t = RC$, i in R-C circuit has decreased $\frac{1}{e}$ its initial value called the relaxation time:

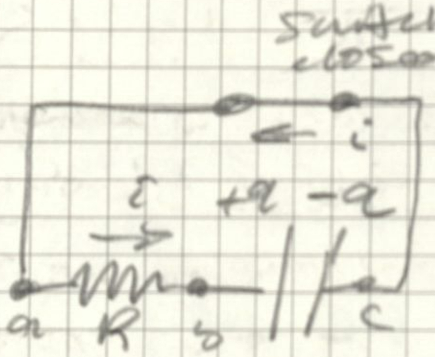
$$\tau = RC$$

* when τ is small, capacitor charges quickly vice versa

Discharging a Capacitor:



capacitor initially charged



discharging the capacitor

$$i = \frac{dq}{dt} = -\frac{q}{RC}$$

* long calculus derivation in book *

$$q = Q_0 e^{-t/RC}$$

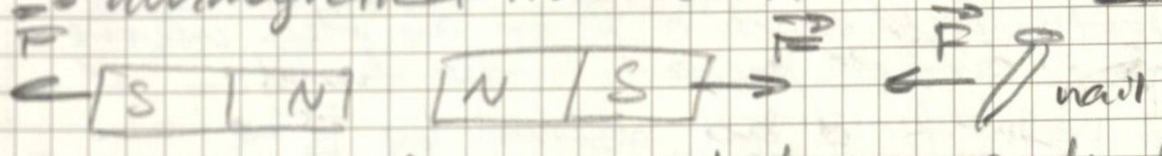
$$i = \frac{dq}{dt} = -\frac{Q_0}{RC} e^{-t/RC} = I_0 e^{-t/RC}$$

* charge in capacitor exponentially decreases as current decreases

Reading Notes: 10.13.24

27.1: Magnetism

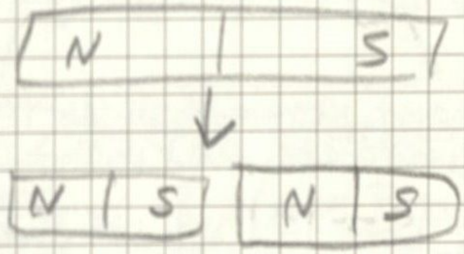
- a bar magnet has a north (N) pole and a south (S) pole
  → when suspended it points N/S (compass)
- opposite poles attract
- like poles repel
- nonmagnetized iron is attracted to *either* pole



- Earth itself is a magnet; has a magnetic field

Magnetic Poles vs Electric Charges:

- poles analogous to pos/neg charges *except*:
  — charges can be isolated
  — magnetic poles *cannot*; always in pairs



breaking a magnet in two

yields two magnets

* magnetic and electrostatic forces can *interact*
— current will deflect a compass needle

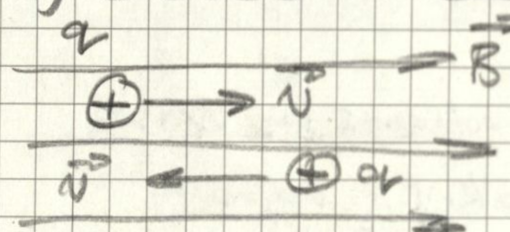
27.2: Magnetic Field

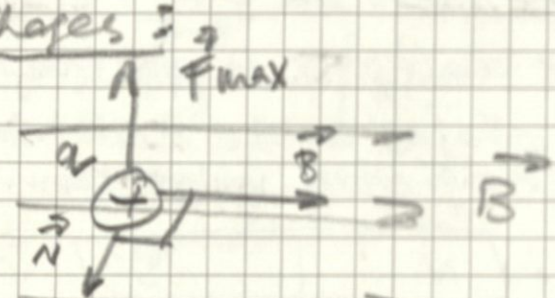
Recall: Electric Field
- distribution of electric charge creates $\vec{E}$ in surrounding space
- $\vec{E}$ exerts a force $\vec{F} = q\vec{E}$ on any q present in $\vec{E}$

Magnetic Field:
- A moving charge or current creates a magnetic field in surrounding space (in addition to its $\vec{E}$)
- Magnetic field exerts a $\vec{F}$ on any other moving charge or current present in the field

* also a vector field and defined by $\vec{B}$ ←
→ at any point on $\vec{B}$, the direction is defined by the direction a compass would point in

Magnetic Forces on Moving Charges:





q moving parallel to $\vec{B}$ experiences $F = 0$

q moving perpendicular to $\vec{B}$ experiences maximum F

$$\vec{F} = q\vec{v} \times \vec{B} \quad (F = |q|v_\perp B = |q|vB\sin\phi)$$

* Follows the RH-Rule (for $q > 0$)
1. curl your fingers in direction of $\vec{v}$ to $\vec{B}$
2. thumb is direction of F
3. vice-versa for negative charges

SI: 1 T (tesla), $1\,T = 1\,N/A \cdot m$

Measuring Magnetic Fields w/ Test Charges:

$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$$

must also account for $\vec{E}$ produced by q

27.3: Magnetic Field Lines and Magnetic Flux

magnetic field lines: same concept and rules as for electric field lines
- except a compass's N/S needle will point in the direction tangent to $\vec{B}$ lines at all points

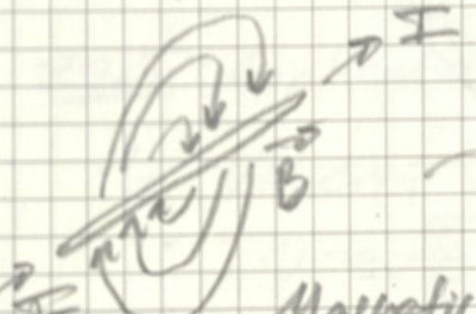
CAUTION: not "lines of force" since:

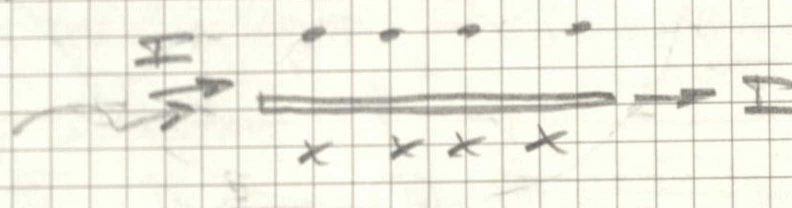
$\vec{F} = q\vec{E}$ and $\vec{F} = q\vec{v} \times \vec{B}$

direct relationship   perpendicular relationship

$\vec{B}$ directed out of paper

$\vec{B}$ directed into paper





Magnetic Flux and Gauss's Law for Magnetism:

$$d\Phi_B = B_\perp dA = B\cos\phi \, dA = \vec{B} \cdot d\vec{A}$$

magnetic flux — same concepts as electric flux!

$$\Phi_B = \int B\cos\phi \, dA = \int B_\perp dA = \int \vec{B} \cdot d\vec{A}$$

SI: "Weber", $1\ Wb = 1\ T \cdot m^2$

Gauss's law for magnetism:

$$\oint \vec{B} \cdot d\vec{A} = 0$$

CAUTION: magnetic field lines have no end! (would imply a monopole)
— continue through interior of magnet

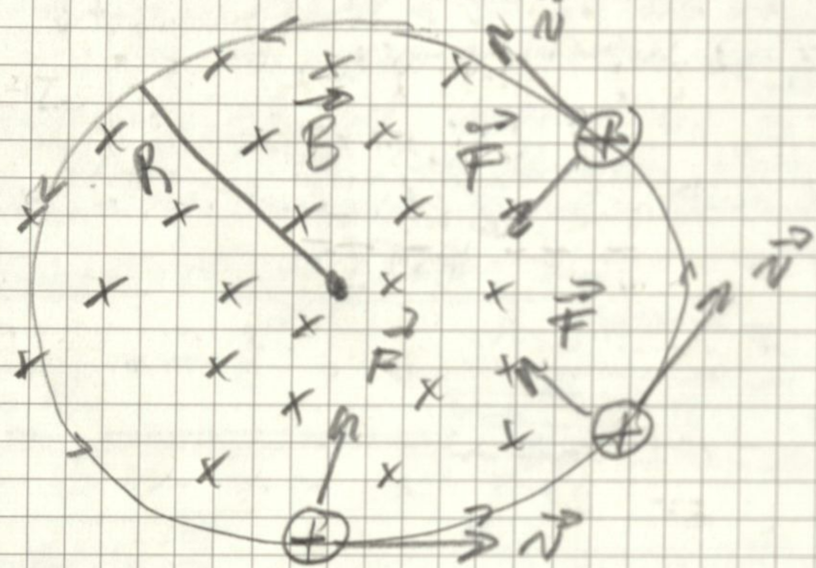
$$B = \frac{d\Phi_B}{dA_\perp}$$ (sometimes called magnetic flux density)

Reading Notes: 10.15.24

27.4: Motion of Charged Particles in a Magnetic Field

since for motion, $\vec{v}$ and $\vec{B}$ are always perpendicular, $\vec{F} = q\vec{v} \times \vec{B}$ has magnitude $F = qvB$ and does no work on the moving particle

→ can only change the direction

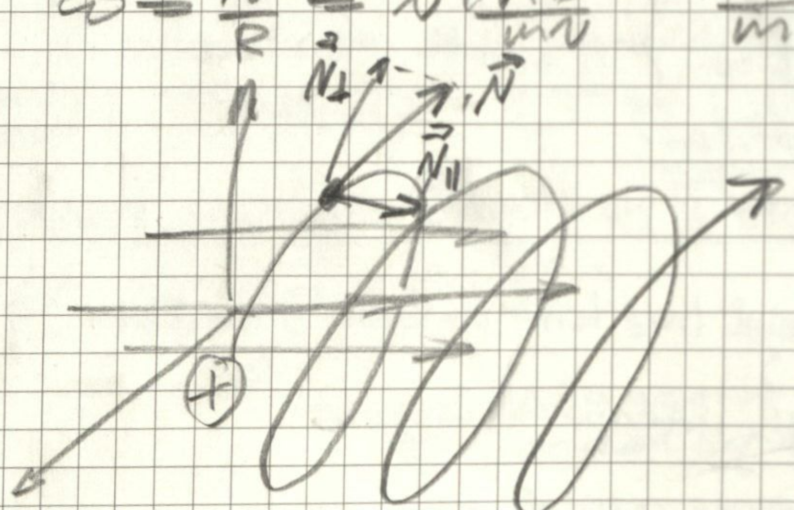


* implies circular motion *

$$F = |q|vB = m\frac{v^2}{R}$$

$$R = \frac{mv}{|q|B}, \quad v = R\omega$$

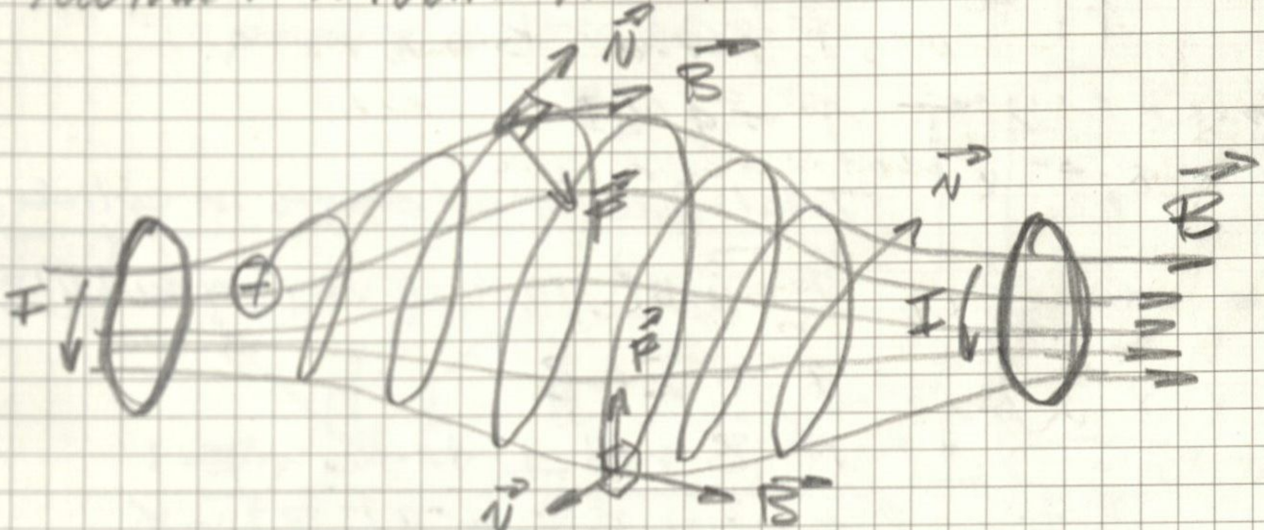
$$\omega = \frac{v}{R} = v\frac{|q|B}{mv} = \frac{|q|B}{m}, \quad f = \frac{\omega}{2\pi}$$



motion has both $v_\parallel$ and $v_\perp$ causing a helical path!

* In a uniform field $\vec{B}$ the field does no work on the particle so its speed and kinetic energy stay constant *

Motion in a NonUniform field is much more complex

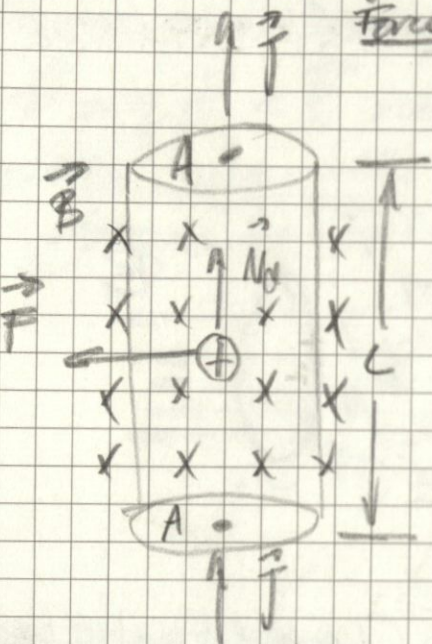


27.5: Applications of Motion of Charged Particles

1. Velocity Selector
$$v = \frac{E}{B}$$
2. Thomson's e/m Experiment
$$\frac{1}{2}mv^2 = eV \quad \text{or} \quad v = \sqrt{\frac{2eV}{m}}$$
3. Mass Spectrometers

Reading Notes: 10.17.24

27.6: Magnetic Force on a Current-Carrying Capacitor



Force for 1 charge:

$$\vec{F} = q\vec{v}_d \times \vec{B}$$

$$F = q v_d B$$

Total force for all charges in cylinder:

$$F = (nAL)(q v_d B) = (nq v_d A)(LB)$$

$$F = ILB$$

if $\vec{B}$ not $\perp$ to the wire:

$$F = ILB_{\perp} = ILB \sin\phi$$

$$\vec{F} = I\vec{L} \times B$$

* same right-hand rule as for moving positive charge *

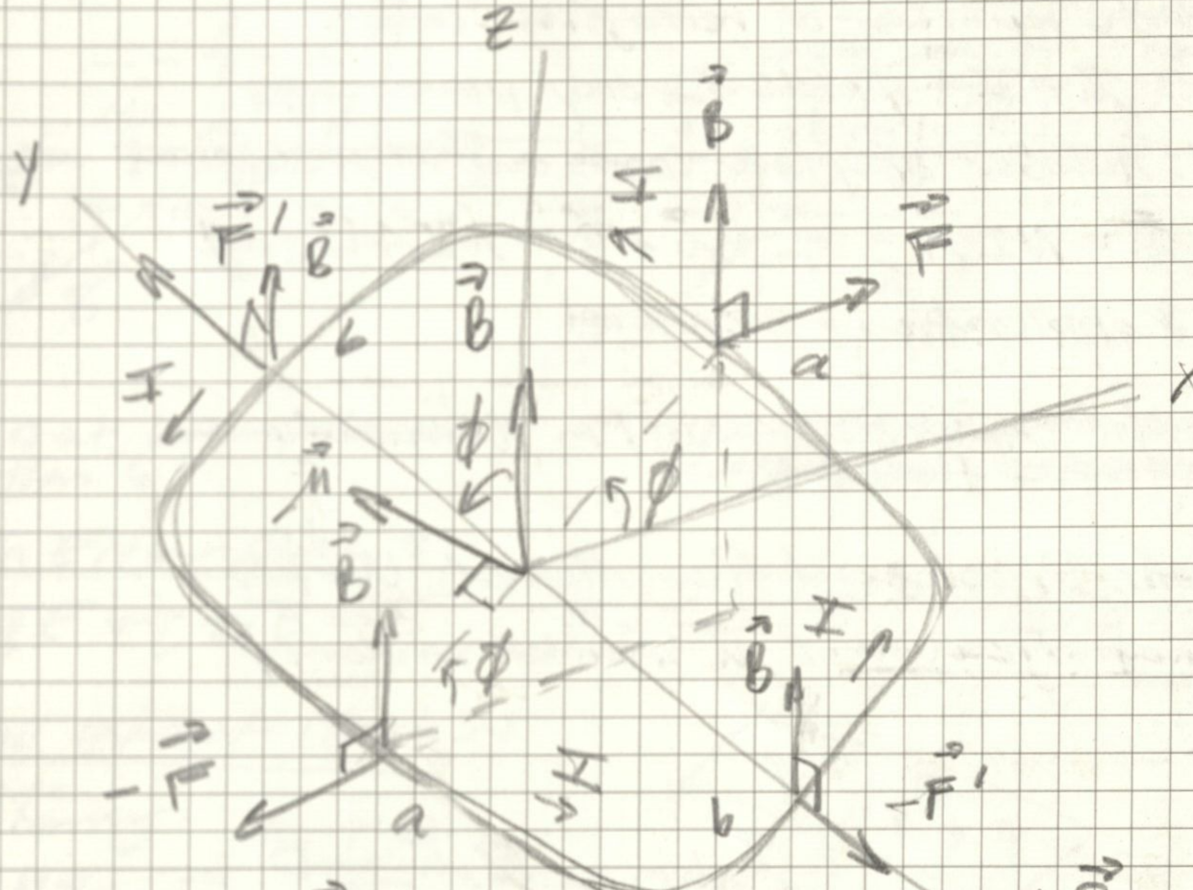
$$d\vec{F} = I\,d\vec{L} \times \vec{B}$$

> so:

- wrap fingers in direction of $\vec{L}$ to $\vec{B}$
- thumb is in direction of $\vec{F}$

27.7 : Force and Torque on a Current Loop

* can treat closed current loops as a series of straight line segments
  - ΣF may be 0 in this case but can produce a $\Sigma \tau \neq 0$



ϕ = angle b/w $\vec{\mu}$ (vector normal to loop) and $\vec{B}$

$\Sigma F = 0$, $\Sigma \tau \neq 0$, $\mu = IA$ ← magnetic dipole moment

radius of loop, force

$\Rightarrow \tau = r \times F = rF\sin\phi = \left(\frac{b}{2}\right) 2F\sin\phi$

$= (IBa)(b\sin\phi) = IBA\sin\phi$ ← area of loop

In Vector Form:

$\vec{\tau} = \vec{\mu} \times \vec{B}$

Potential Energy for a Magnetic Dipole:

$$U = -\vec{\mu} \cdot \vec{B} = -\mu B \cos\phi$$

Magnetic Torque: Loops and Coils:

* any planar loop can be approximated with a large number of rectangular loops

$\vec{\mu} = I\vec{A}$ holds for any planar loop

N: number of planar loops current makes

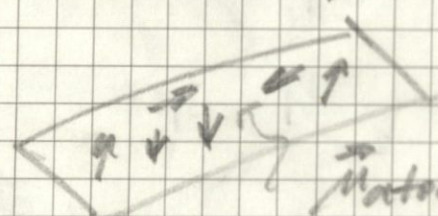
$\vec{\mu} = NI\vec{A}$, $\vec{\tau} = \vec{\mu} \times \vec{B} = NIAB\sin\phi$

* application: solenoid

* Magnetic Dipoles also exist in nonuniform magnetic fields *

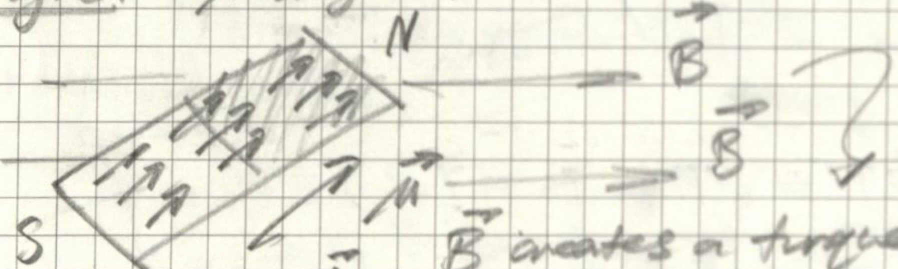
Magnetic Dipoles and How Magnets Work:

unmagnetized iron: $\vec{\mu}$ oriented randomly

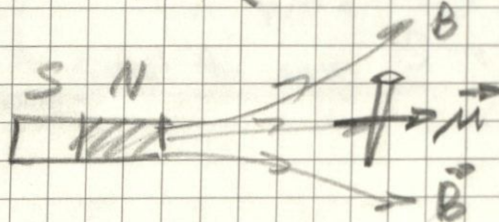


$\vec{\mu}_{atom}$, $\Sigma\vec{\mu} = \vec{0}$

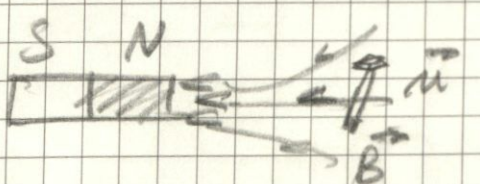
bar magnet: $\vec{\mu}$ aligned



$\vec{B}$ creates a torque in the bar magnet aligning its $\vec{\mu}$ with $\vec{B}$



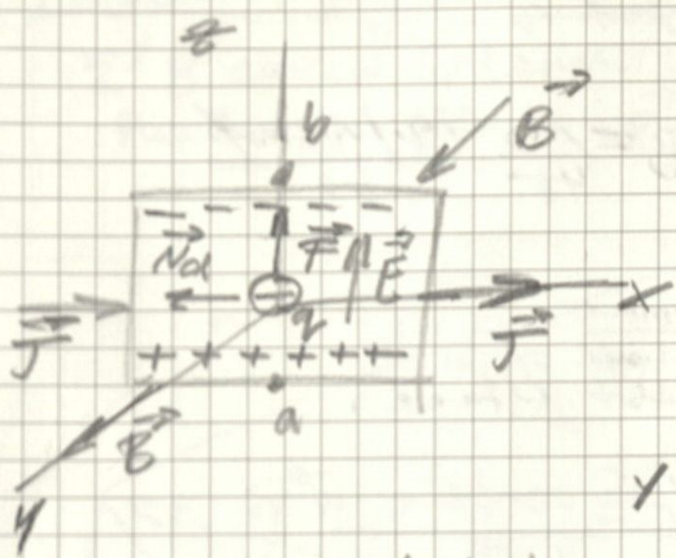
nail is repelled



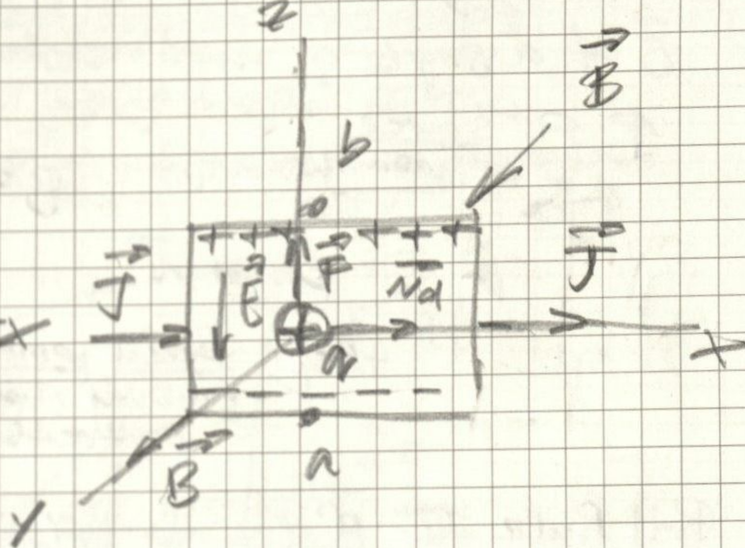
nail is attracted

Reading Notes: 10.27.24

27.9: The Hall Effect



a is at higher potential than b



polarity of potential difference is opposite

In Either Case:

$$qE + q v_d B = 0, \quad E = -v_d B, \quad J = nq v_d$$

Hall Effect (Book):

$$nq = \frac{-JB}{E}$$

charge per carrier

concentration of moving charge carriers

electrostatic field inside conductor

Hall Effect (formula sheet):

$$V = \frac{IB}{nte}$$

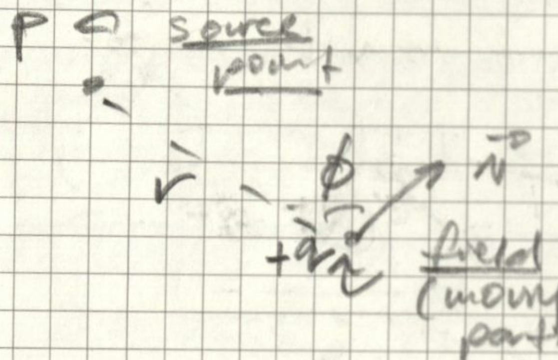
charge of carrier (often electrons)

thickness of conductor

Reading Notes: 10.30.24

28.1: Magnetic Field of a Moving Charge

$\vec{B}$ of a single q with constant $\vec{v}$

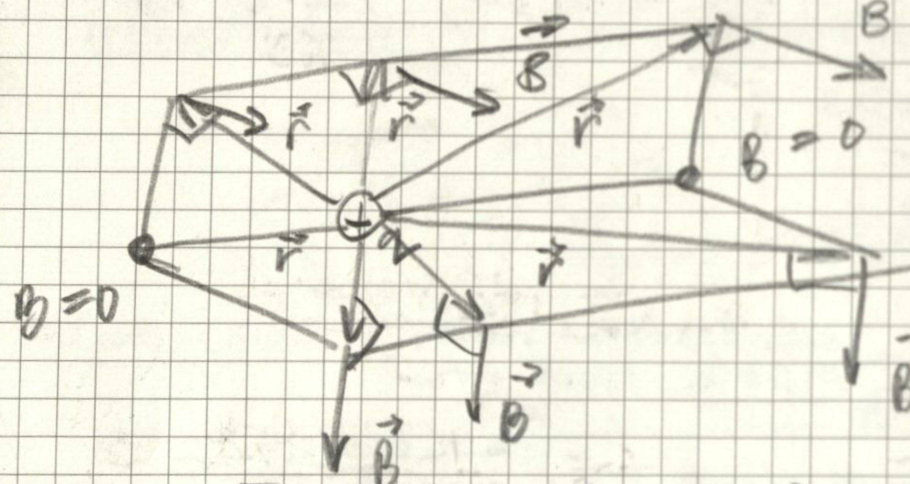


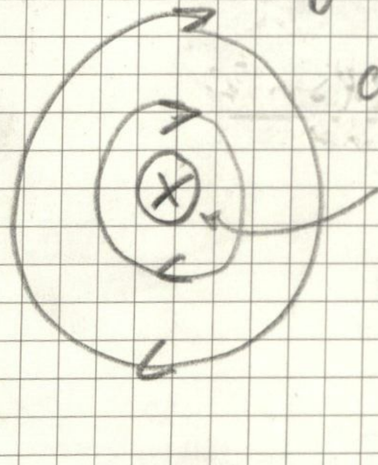
$$B = \frac{\mu_0}{4\pi}\frac{|q|v\sin\phi}{r^2}.$$

R-H Rule for $\vec{B}$ due to $+q$ @ constant $\vec{v}$:

1. Point thumb in direction of $\vec{v}$
2. fingers curl around $+q$ in direction of $\vec{B}$

* opposite for $-q$





$$\vec{B} = \frac{\mu_0}{4\pi}\frac{q\vec{v}\times\hat{r}}{r^2}$$

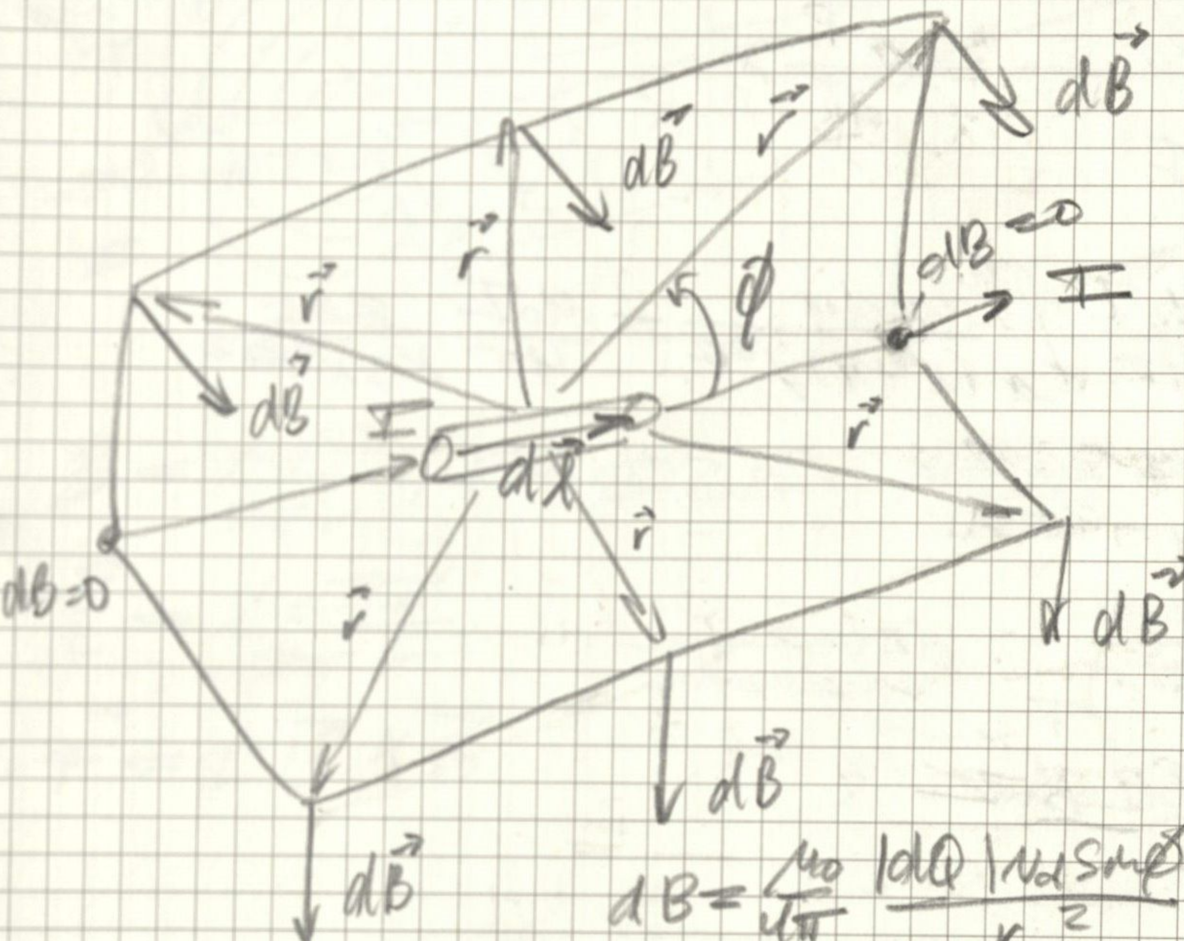
μ_0: magnetic constant

SI: T · m/A

28.2: Magnetic Field of a Current Element

Principle of Superposition of Magnetic Fields:

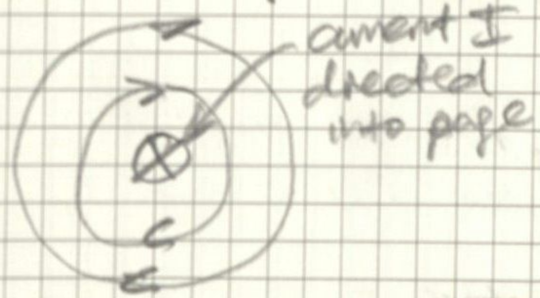
$\Sigma\vec{B}$ caused by several moving charges is the vector sum of the $\vec{B}_i$ of individual charges



$$dB = \frac{\mu_0}{4\pi}\frac{|dQ|\,v_d \sin\phi}{r^2}$$

$$= \frac{\mu_0}{4\pi}\frac{n|q|v_d A\,dl \sin\phi}{r^2}$$

$$dB = \frac{\mu_0}{4\pi}\frac{I\,dl \sin\phi}{r^2}$$



Current Element: Vector $\vec{B}$

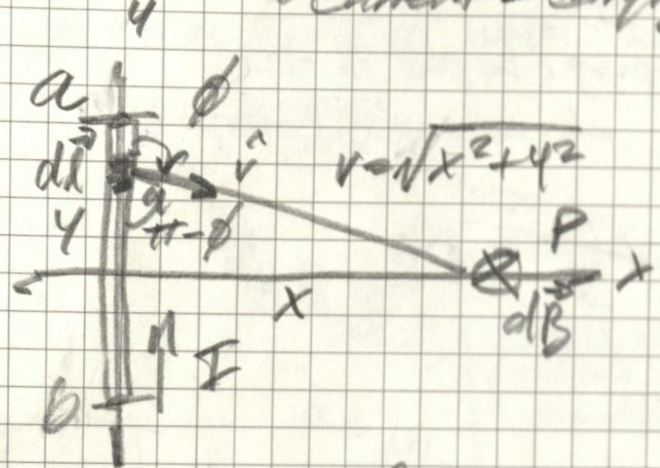
$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{I\,d\vec{l}\times\hat{r}}{r^2}$$

$$\vec{B} = \frac{\mu_0}{4\pi}\int\frac{I\,d\vec{l}\times\hat{r}}{r^2}$$

Reading Notes:

10.31.24

28.3: Magnetic Field of a Straight Current-carrying Conductor



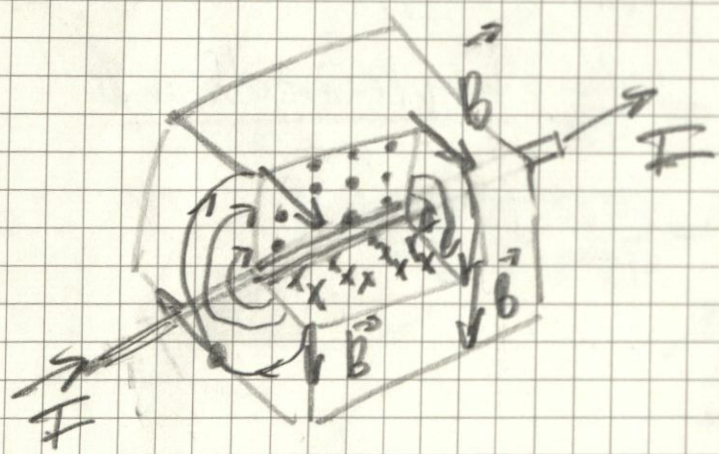
$$B=\frac{\mu_0 I}{4\pi}\int_{-a}^{a}\frac{x\,dy}{(x^2+y^2)^{3/2}}=\frac{\mu_0 I}{4\pi}\,\frac{2a}{x\sqrt{x^2+a^2}}$$

$$\lim_{a\to\infty} B=\frac{\mu_0 I}{2\pi x}$$

$$B=\frac{\mu_0 I}{2\pi r}$$

distance from conductor

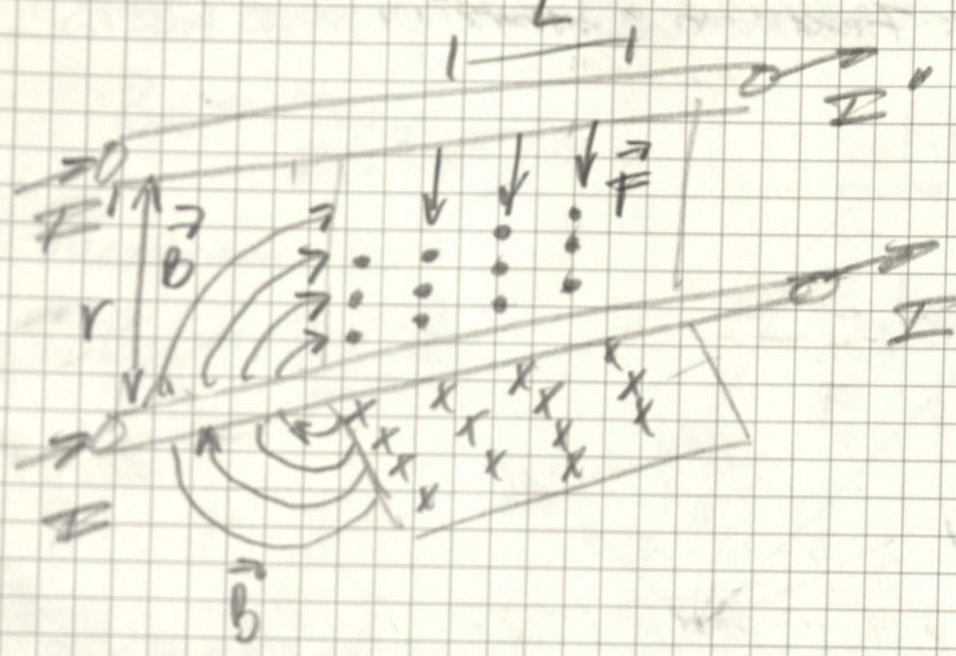
Follows RH Rule:



* Also, all $\vec{B}$ to enter a surface must exit that surface *

$$\oint \vec{B}\cdot d\vec{A}=0$$

28.4: Force Between Parallel Conductors 10.31.24



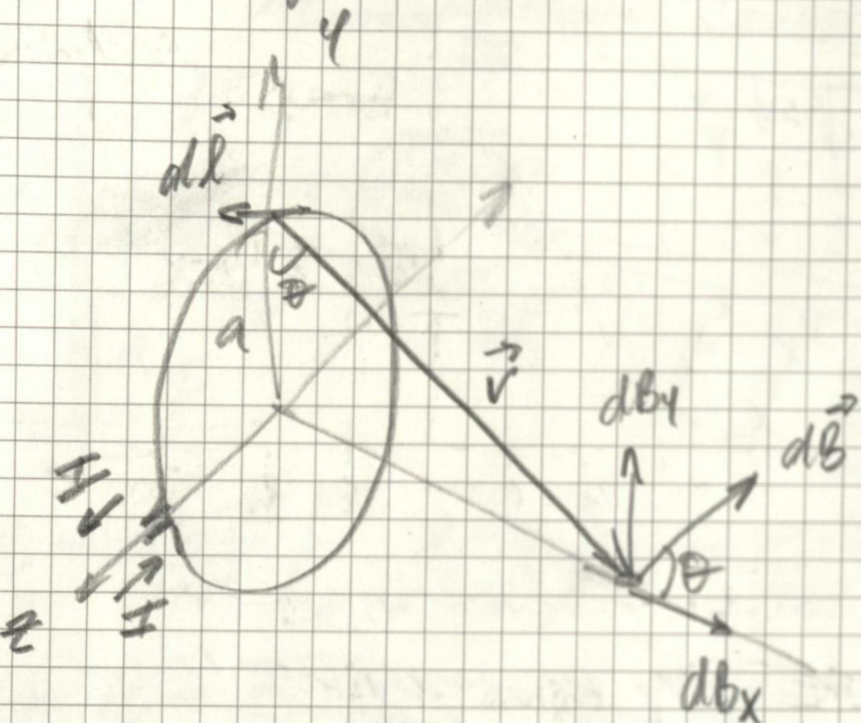
$$B = \frac{\mu_0 I}{2\pi r}, \quad F = I' L B = \frac{\mu_0 I I' L}{2\pi r}$$

$$\frac{F}{L} = \frac{\mu_0 I I'}{2\pi r}$$

$$\vec{F} = I \vec{L} \times \vec{B}$$

Reading Notes: 11.5.24

28.5: Magnetic Field of a Circular Current Loop



$$dB = \frac{\mu_0 I}{4\pi}\frac{dl}{(x^2+a^2)}, \quad dB_x = dB\cos\theta$$
$$dB_y = dB\sin\theta$$

* total field $\vec{B}$ only has an x-component because of symmetry

$$B_x = \int \frac{\mu_0 I}{4\pi}\frac{a\,dl}{(x^2+a^2)^{3/2}} = \frac{\mu_0 I a}{4\pi(x^2+a^2)^{3/2}}\int dl$$

$$B_x = \frac{\mu_0 I a^2}{2(x^2+a^2)^{3/2}}$$

* direction given by RH-rule
  - fingers curl in direction of I
  - thumb points in direction of $\vec{B}$

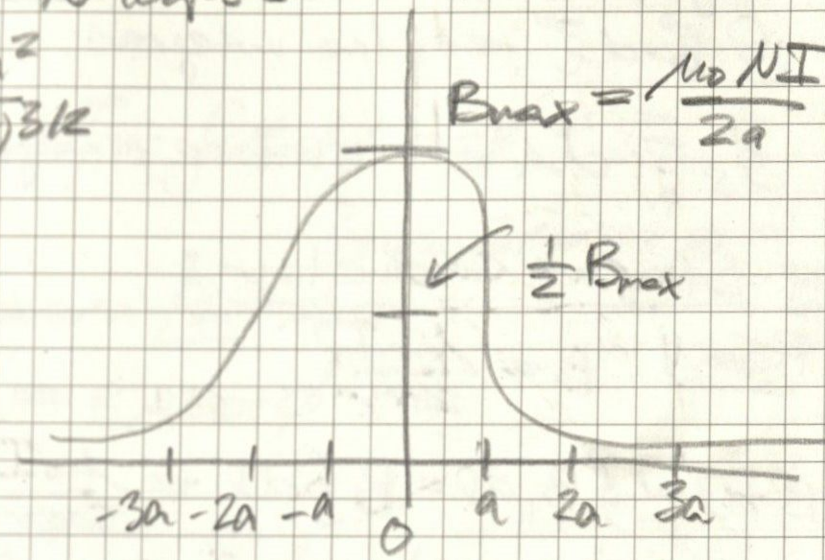
Magnetic Field on the Axis of a Coil:

a coil consisting of N loops:

$$B_x = \frac{\mu_0 N I a^2}{2(x^2+a^2)^{3/2}}$$

Maximum B_x:

$$B_x = \frac{\mu_0 N I}{2a}$$



on the axis of any number of circular loops:

$$B_x = \frac{\mu_0 \mu}{2\pi(x^2+a^2)^{3/2}}$$ magnetic dipole moment

* all still derived from Biot-Savart *

→ analogous to Coulomb's Law

28.6: Ampere's Law

analogous to Gauss's Law except:

$\oint \vec{B}\cdot d\vec{\ell}$ ← line integral

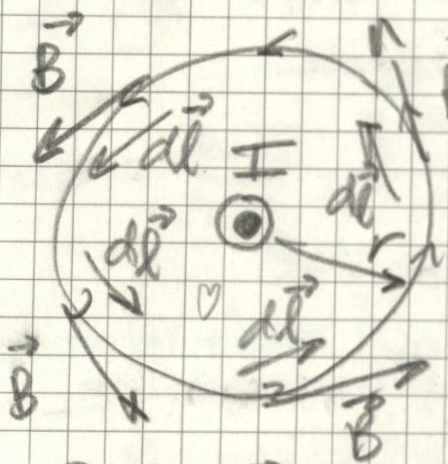
$\oint_S \vec{E}\cdot d\vec{A}$ ← surface integral

Long-Straight Conductor:

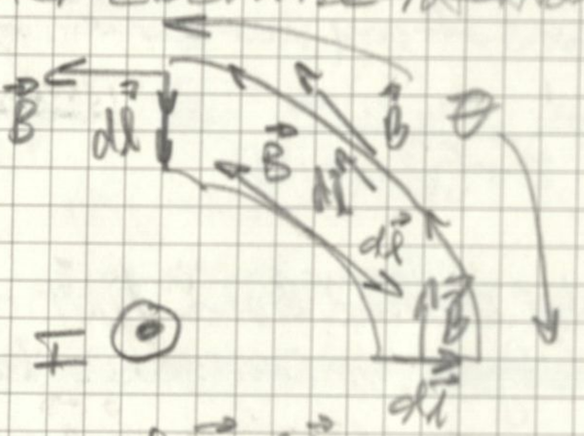
Recall: $B = \dfrac{\mu_0 I}{2\pi r}$

$$\oint \vec{B}\cdot d\vec{\ell} = \oint B_{\parallel}\, d\ell = B\oint d\ell = \frac{\mu_0 I}{2\pi r}(2\pi r) = \mu_0 I$$

* conventionally w/ counter-clockwise rotation of I *



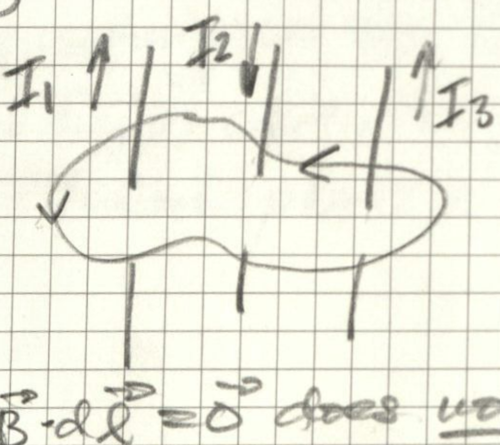
$\oint \vec{B}\cdot d\vec{\ell} = \mu_0 I$

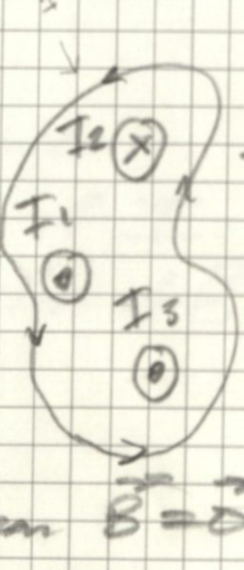


$\oint \vec{B}\cdot d\vec{\ell} = 0$

General Statement:

$$\oint \vec{B}\cdot d\vec{\ell} = \mu_0 I_{encl}$$





$I_{encl} = I_1 - I_2 + I_3$

* $\oint \vec{B}\cdot d\vec{\ell} = \vec{0}$ does not mean $\vec{B} = \vec{0}$ everywhere along the path!

Reading Notes: 11/1.24

29.2. Faraday's Law

Magnetic Flux:

$$\Phi_B = \int \vec{B} \cdot d\vec{A} = \int B\,dA\cos\phi$$

angle btw $\hat{n}$ and $\vec{B}$

if $\vec{B}$ is uniform over a flat A

$$\Phi_B = \vec{B} \cdot \vec{A} = BA\cos\phi$$

Faraday's Law:

$$\mathcal{E} = -\frac{d\Phi_B}{dt}$$

induced emf of closed loop

negative time rate of magnetic flux through the loop

Direction of induced emf:

1. define a positive direction for $\vec{A}$
2. from the direction of $\vec{A}$ and $\vec{B}$, determine the sign of Φ_B and $d\Phi_B/dt$
3. the direction of emf is opposite of that

Reading Notes: 11.10.24

29.3: Lenz's Law

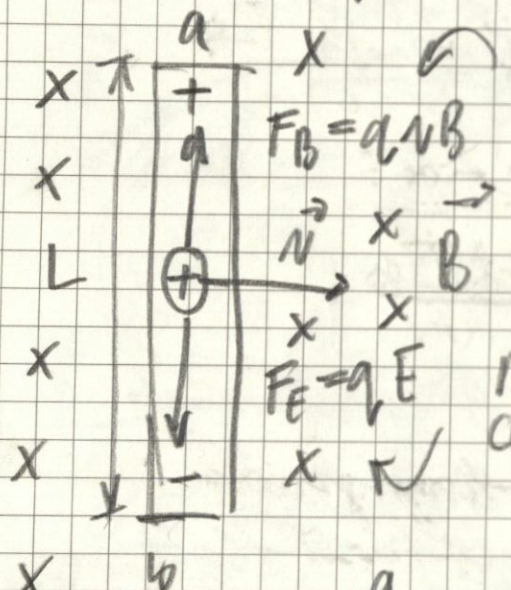
not an independent principle — derived from Faraday's Law

Lenz's Law: the direction of any magnetic induction effect is opposite to the cause of that effect

ex: changing Φ_B due to varying $\vec{B}$ through a circuit

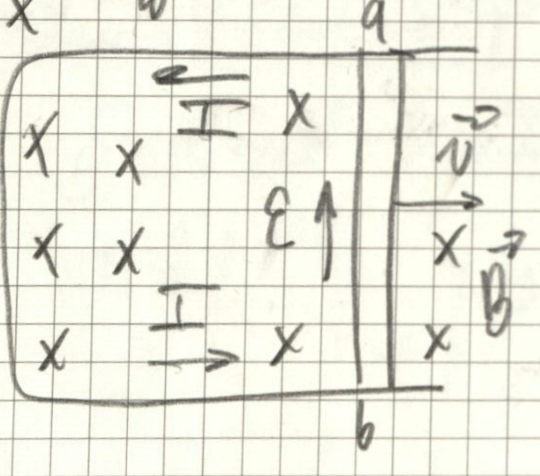
29.4: Motional emf

Recall: $\vec{F} = q\vec{v} \times \vec{B}$, $F = |q|vB$



charges in the moving rod are acted upon F_B

resulting charge separation causes canceling electric force F_E



$V_{ab} = EL = vBL$

$\mathcal{E} = vBL$

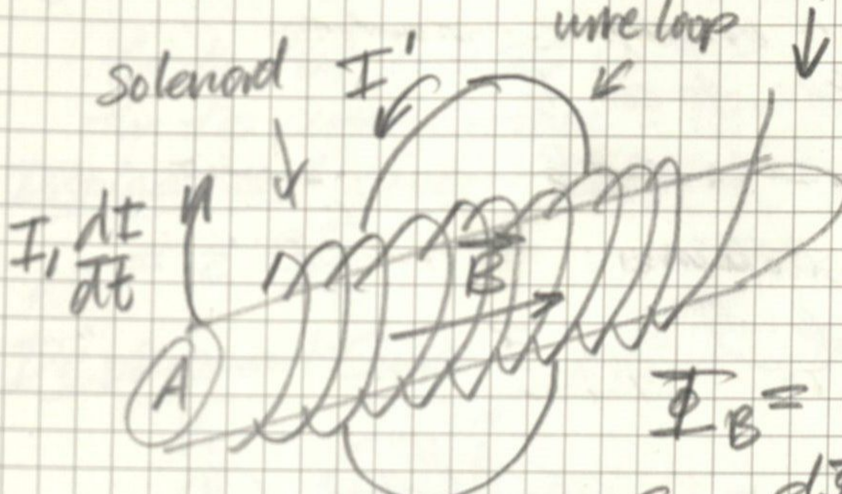
General Form:

$$d\mathcal{E} = (\vec{v} \times \vec{B}) \cdot d\vec{l}$$

$$\mathcal{E} = \oint (\vec{v} \times \vec{B}) \cdot d\vec{l}$$

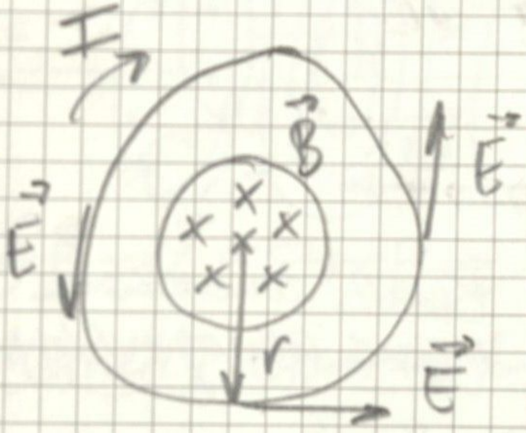
Reading Notes: 11.12.24

29.5: Induced Electric Fields



$$\Phi_B = BA = \mu_0 n I A$$

$$\mathcal{E} = -\frac{d\Phi_B}{dt} = -\mu_0 n A \frac{dI}{dt}$$



$$\mathcal{E} = \oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt}$$

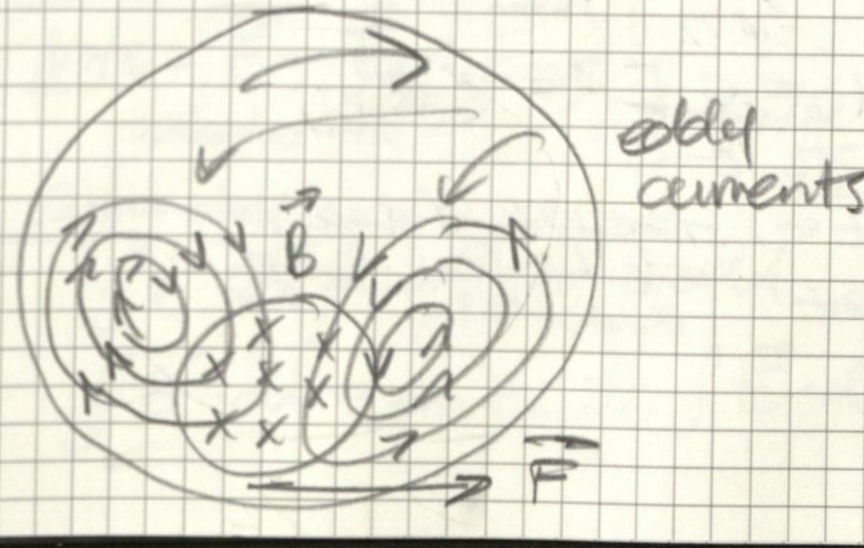
For Circular Loops:

$$E = \frac{1}{2\pi r}\left|\frac{d\Phi_B}{dt}\right|$$

29.6 - Eddy Currents

"induced currents that circulate throughout the volume of a material instead of well-defined paths of conductors"

Ex: metal disk rotating through a $\vec{B}$



Reading Notes: 11.14.24

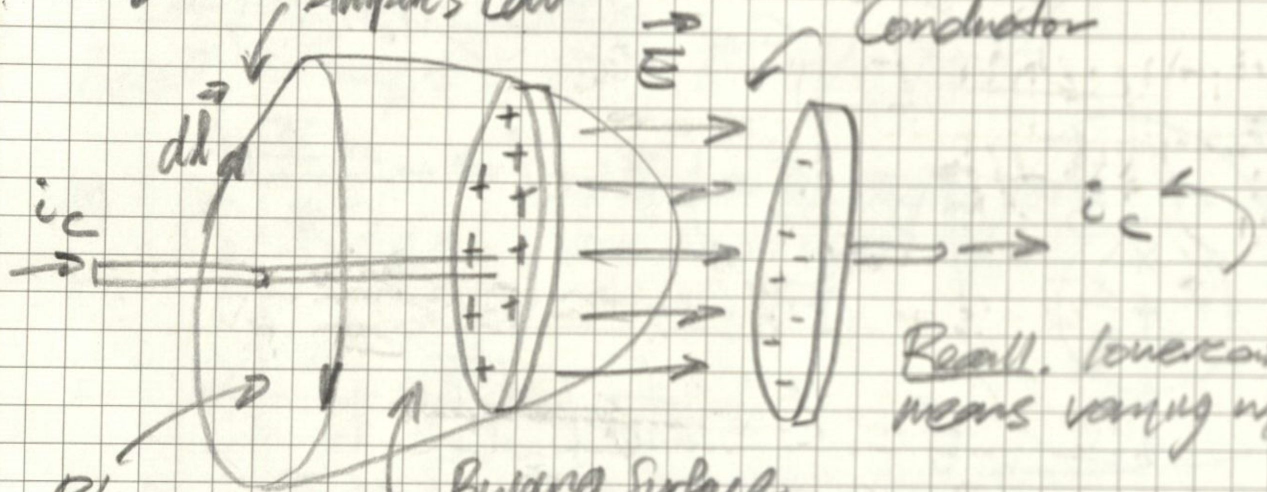
29.7: Displacement Currents and Maxwell's Equations

Symmetry:
- varying $\vec{B}$ gives rise to an $\vec{E}$
- varying $\vec{E}$ gives rise to a $\vec{B}$

Recall: $\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{encl}$

not the full story

Imagine:



Recall: lowercase means varying w/ time

$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{encl} = \mu_0 i_C$ for planar surface

$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{encl} = 0$ for bulging surface

$\therefore$ contradiction!

Conductor Properties:

$q = Cv = \frac{\varepsilon A}{d}(Ed) = \varepsilon EA = \varepsilon \Phi_E$

$i_D = \frac{dq}{dt} = \varepsilon \frac{d\Phi_E}{dt}$ time rate of change of electric flux through area

displacement current through an area (conductor)

Generalized Ampere's law

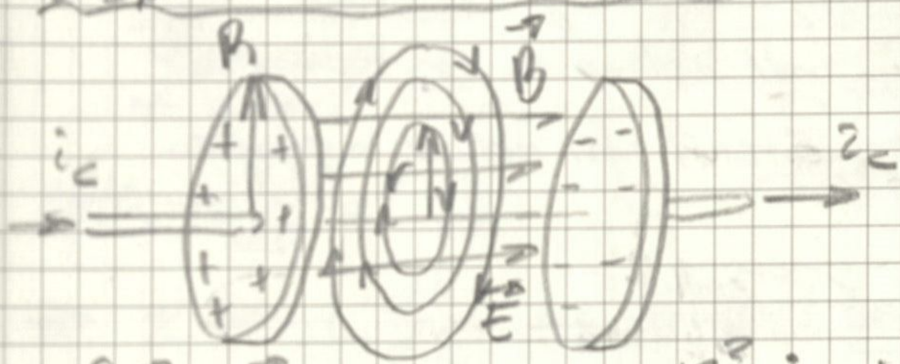
$$\oint \vec{B} \cdot d\vec{l} = \mu_0 (i_C + i_D)_{encl}$$

for the flat surface: $i_D = 0$
for the curved surface: $i_C = 0$

Displacement Current Density:

$$j_D = \varepsilon \frac{dE}{dt}$$

Displacement Current $\oint \vec{B}$:



$$\oint \vec{B} \cdot d\vec{l} = 2\pi r B = \mu_0 \frac{r^2}{R^2} i_C \Rightarrow B = \frac{\mu_0}{2\pi} \frac{r}{R^2} i_C$$

Maxwell's Equations for Electromagnetism:

Gauss's law for $\vec{E}$: $\oint \vec{E} \cdot d\vec{A} = \frac{Q_{encl}}{\varepsilon_0}$

Gauss's law for $\vec{B}$: $\oint \vec{B} \cdot d\vec{A} = 0$

Faraday's law: $\oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt}$

Ampere's law: $\oint \vec{B} \cdot d\vec{l} = \mu_0 \left(i_C + \varepsilon_0 \frac{d\Phi_E}{dt}\right)_{encl}$

Note: $\vec{E} = \vec{E}_c + \vec{E}_n$

electrostatic   magnetically induced

Symmetry:

$$\oint \vec{E} \cdot d\vec{l} = -\frac{d}{dt} \int \vec{B} \cdot d\vec{A}$$

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 \varepsilon_0 \frac{d}{dt} \int \vec{E} \cdot d\vec{A}$$

Reading Notes: 11.17.24

30.1: Mutual Inductance

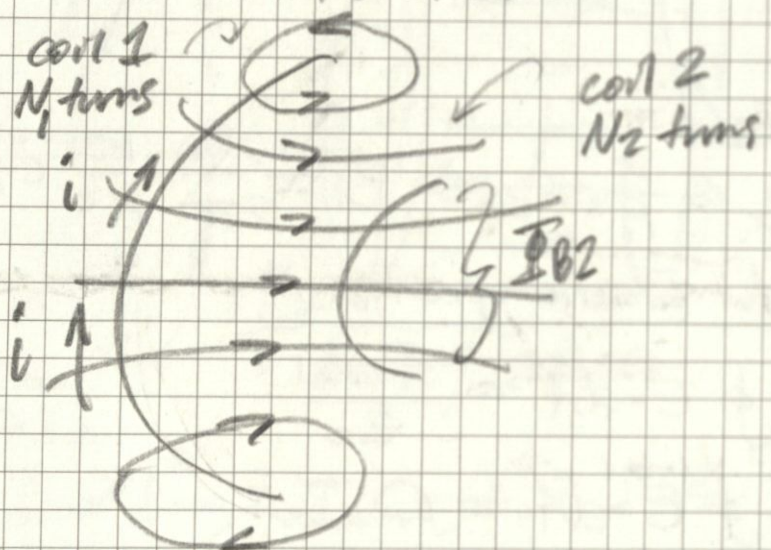
Recall, two wires carrying steady currents:

1. current in one wire causes a $\vec{B}$
2. $\vec{B}$ causes a force on the current in the second wire

Now, two coils carrying changing currents:

1. coil 1 will produce a $\vec{B}$ and hence a Φ_B in coil 2
2. as Φ_{B1} changes Φ_{B2} will change as well
3. this change in Φ_{B2} will induce a $\mathcal{E}$ in coil 2

$\therefore$ change in current in one circuit will induce a change in current in the other



$$\mathcal{E}_2 = -M\frac{di_1}{dt} \quad \text{and} \quad \mathcal{E}_1 = -M\frac{di_2}{dt}$$

mutual inductance of coils 1 and 2

$$M = \frac{N_2 \Phi_{B2}}{i_1} = \frac{N_1 \Phi_{B1}}{i_2}$$

SI: 1 henry (1 H) = 1 Wb/A

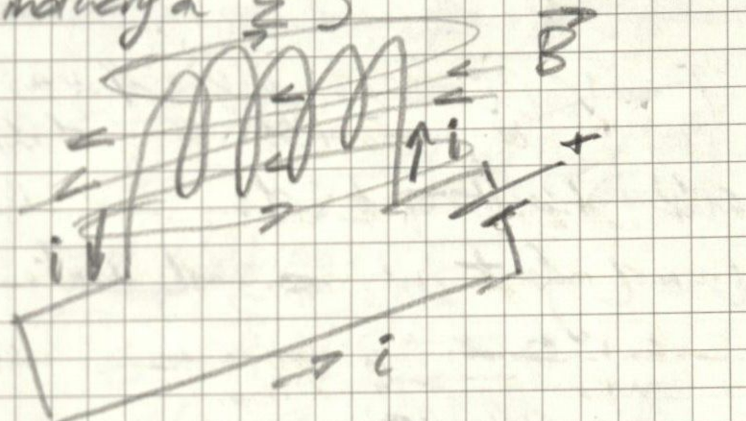
30.2. Self Inductance and Inductors

Recall. Mutual Inductance (left)

Now, the same effect actually happens in a single isolated circuit

(a change in current creates a $\Delta \vec{B}$ that causes a $\Delta \Phi_B$ in the same circuit, inducing a $\mathcal{E}$)

$$L = \frac{N \Phi_B}{i}$$



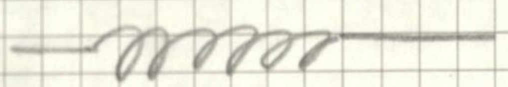
self inductance (inductance) of a coil

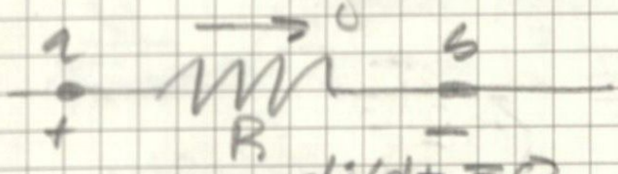
Note. $N \frac{d\Phi_B}{dt} = L \frac{di}{dt}$

$\Rightarrow \mathcal{E} = -L \frac{di}{dt}$ by Faraday's Law

Inductors as Circuit Elements:

- circuit device that has a particular inductance
- called an inductor, or choke

Symbol: 



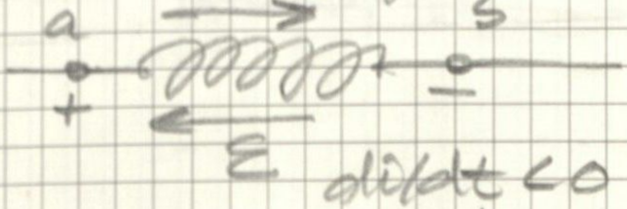
$V_{ab} = iR > 0$

$di/dt = 0$

$V_{ab} = L \frac{di}{dt} = 0$

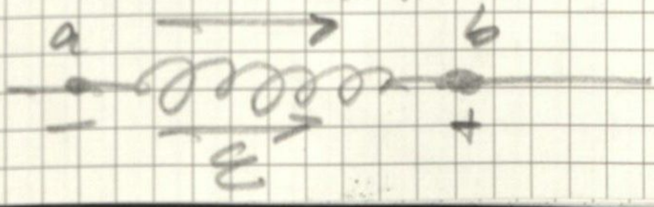
$\mathcal{E} = 0$

$di/dt > 0$



$V_{ab} = L \frac{di}{dt} > 0$

$di/dt < 0$



$V_{ab} = L \frac{di}{dt} < 0$

Reading Notes: 11.19.24

30.3 Magnetic-Field Energy

Energy Stored in an Inductor:

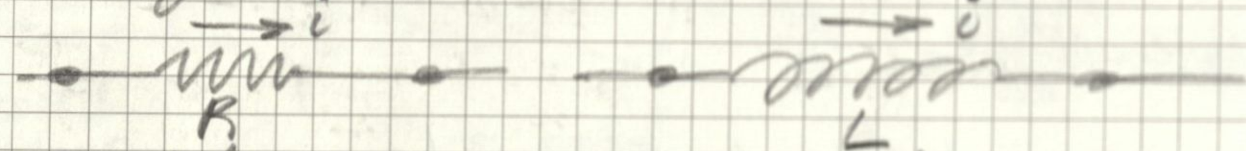
To ~~calculate total~~ energy U needed to establish a final current I in and inductor w/ inductance L if $I_0 = 0$. Assume no resistance and that $\frac{di}{dt} > 0$

$$P = V_{ab}\, i = L i \frac{di}{dt}$$ (rate P of energy being delivered to inductor)

$$dU = P\,dt = L i\,di$$

$$U = L \int_0^I i\,di = \frac{1}{2} L I^2$$

Note.



energy is being dissipated

energy is being stored

Magnetic Energy Density:

inductor → energy stored in $\vec{B}$ of the coil

conductor → energy stored in $\vec{E}$ between the plates

Note.

$$L = \frac{\mu_0 N^2 A}{2\pi r}, \quad U = \frac{1}{2} L I^2, \quad u = \frac{U}{V} = \frac{U}{2\pi r A}$$

inductance — energy — energy density

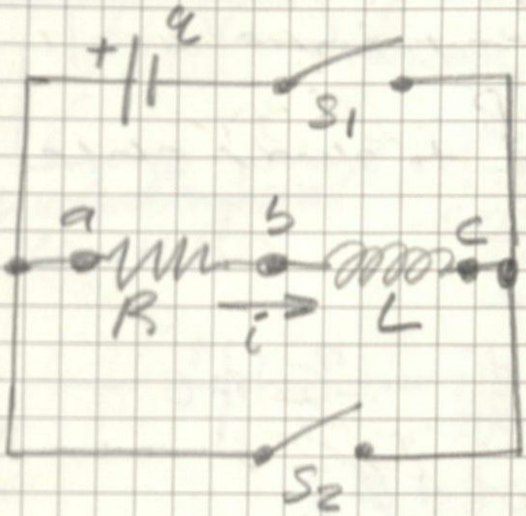
when used together,

$$u = \frac{B^2}{2\mu_0} \quad \text{or} \quad u = \frac{B^2}{2\mu}$$

permeability of material

30.4: The R-L Circuit

- an inductor in a circuit makes it difficult for rapid changes in current to occur
- an R-L circuit is a circuit that has a resistor and an inductor



Both switches open, @ $t=0$ close S_1:

$$v_{ab} = iR \text{ and } v_{bc} = L\frac{di}{dt}$$

$\sum_{loop} \Delta V = 0$:

$$\mathcal{E} - iR - L\frac{di}{dt} = 0$$

$$\Rightarrow \frac{di}{dt} = \frac{\mathcal{E} - iR}{L} = \frac{\mathcal{E}}{L} - \frac{iR}{L}$$

Note: $\left(\frac{di}{dt}\right)_{initial} = \frac{\mathcal{E}}{L}$

* the greater the L, the slower the current increases *

And: $\left(\frac{di}{dt}\right)_{final} = 0 = \frac{\mathcal{E}}{L} - \frac{R}{L}I \Rightarrow I = \frac{\mathcal{E}}{R}$

Time constant: how quickly current builds to final value

$$\tau = \frac{L}{R}$$

* energizing and de-energizing current formulas derived in textbook *

Reading Notes: 11.21.24

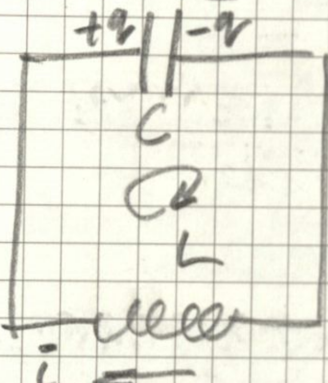
30.5: The L-C Circuit

Recall: R-C Circuit → "registor - capacitor"
- stores energy in an $\vec{E}$
- exponential approach to steady-state

R-L Circuit → "registor - inductor"
- stores energy in a $\vec{B}$
- exponential approach to steady state

L-C Circuit: "inductor - capacitor"

oscillates current and charge



$$\sum V_{loop} = -L\frac{di}{dt} - \frac{q}{C} = 0$$

$$q = Q\cos(\omega t + \phi)$$

$$\omega = \sqrt{\frac{1}{LC}}$$

angular frequency of oscillation

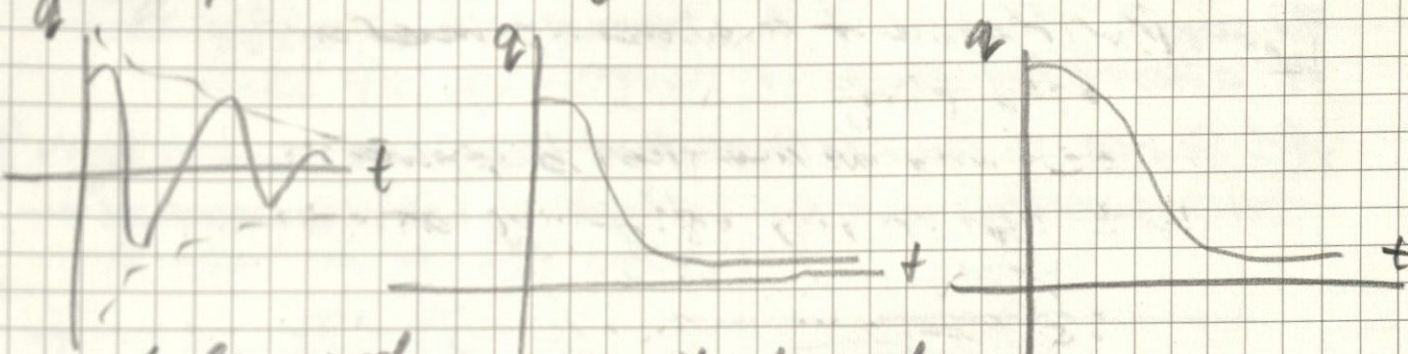
$$i = -\omega Q \sin(\omega t + \phi)$$

Energy in an L-C Circuit:

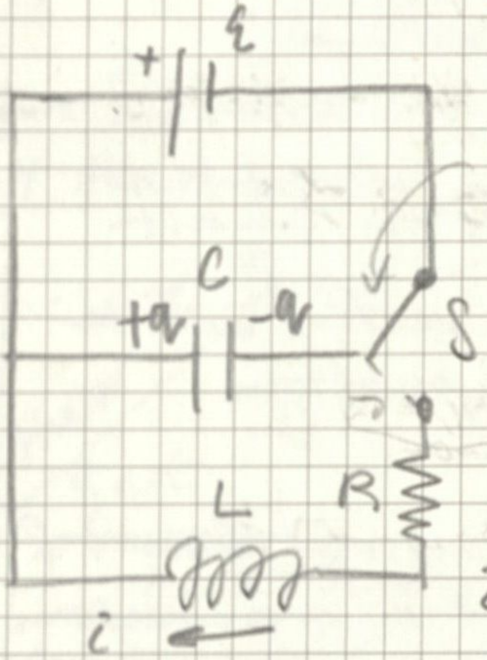
Mass-Spring System	Inductor-Capacitor Circuit
$K = \frac{1}{2}mv_x^2$	$U_L = \frac{1}{2}Li^2$
$U = \frac{1}{2}kx^2$	$U_C = q^2/2C$
$K + U = \frac{1}{2}kA^2$	$U_C + U_L = Q^2/2C$
$v_x = \pm\sqrt{k/m}\sqrt{A^2 - x^2}$	$i = \pm\sqrt{1/LC}\sqrt{Q^2 - q^2}$
$v_x = dx/dt$	$i = dq/dt$
$\omega = \sqrt{\frac{k}{m}}$	$\omega = \sqrt{\frac{1}{LC}}$
$x = A\cos(\omega t + \phi)$	$q = Q\cos(\omega t + \phi)$

30.6: The L-R-C Series Circuit

dampened oscillating circuit



underdampened (small R) | critically damped (larger R) | overdampened (very large R)



when switch S in this position, emf charges the capacitor

when switch S in this position, capacitor discharges through resistor and inductor

$$\sum V_{loop} = -iR - L\frac{di}{dt} - \frac{q}{C} = 0$$

$$q = Ae^{-(R/2L)t}\cos\left(\sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}\,t + \phi\right)$$

$$\omega' = \sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}$$

underdampened diffeq solutions

* solutions different in critically damped and overdampened situations

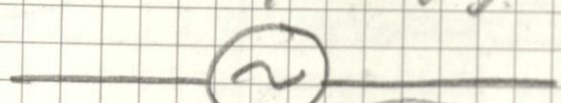
Reading Notes:

11.24.24

31.1: Phasors and Alternating Currents

To supply AC current to a circuit, need a source of alternating emf

AC Source: any device that supplies a sinusoidally varying voltage v or current i



$v = V \cos \omega t$

angular frequency $2\pi f$

maximum potential difference (voltage amplitude)

instantaneous potential diff

$i = I \cos \omega t$

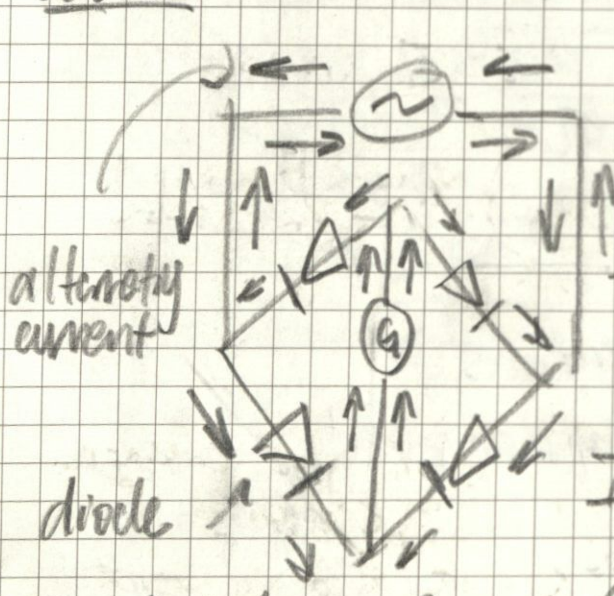
same relationships but for current

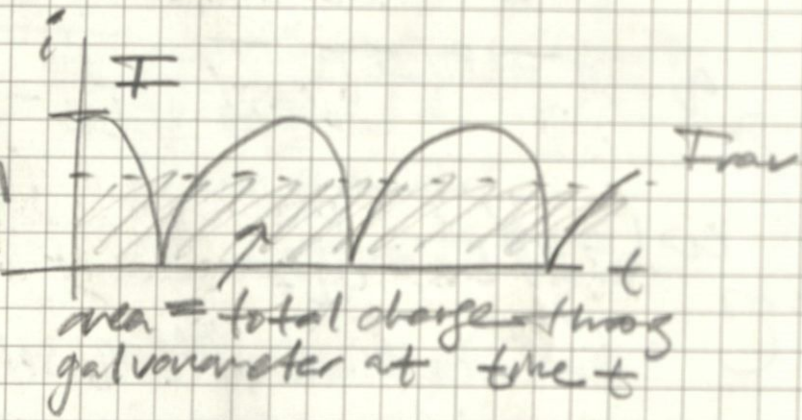
Phasor Diagrams: rotating vectors similar to those in simple harmonic motion

Rectified Alternating Current:

* galvanometers cannot measure sinusoidal current *

diode: used to measure current in one direction





$I_{rav} = \frac{2}{\pi} I$

Rectified Average Current (I_{rav}): total charge that flows through the circuit is equal to I_{rav} and the same as if the current was constant

Root-Mean-Square (rms) Values:

I_{rms}: square the instantaneous current i
take the average of i^2
take the square root of that average

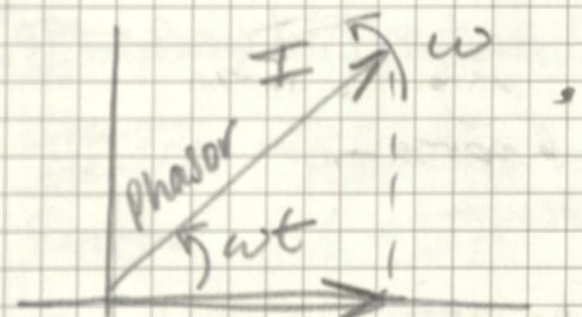
& rms values are **never** negative
→ represents the "effective" current equivalent to that in a DC circuit

$$I_{rms} = \frac{I}{\sqrt{2}} \qquad V_{rms} = \frac{V}{\sqrt{2}}$$

current amplitude — voltage amplitude

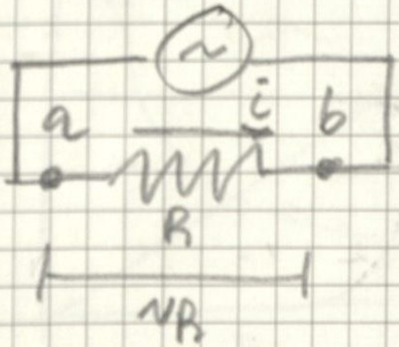
31.2: Resistance and Reactance

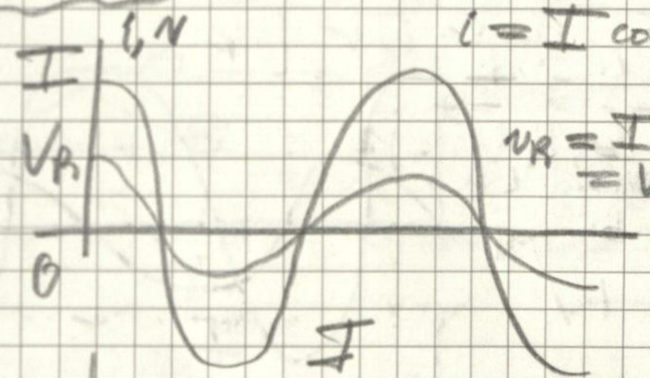
Recall Phasors



- length of phasor is maximum current I
- phasor rotates with frequency f and angular speed $\omega = 2\pi f$
- projection of phasor onto horizontal axis at time t is current i at that instance

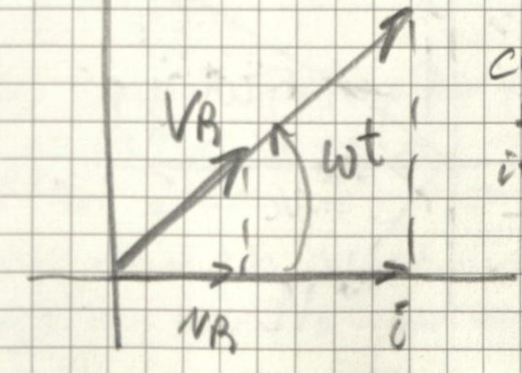
Resistor in an ac Circuit:





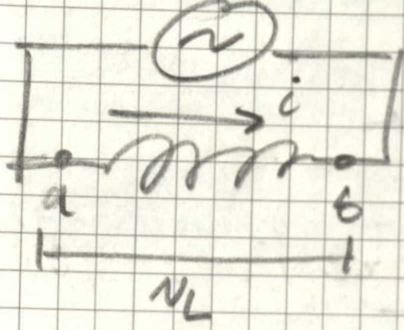
$i = I\cos\omega t$

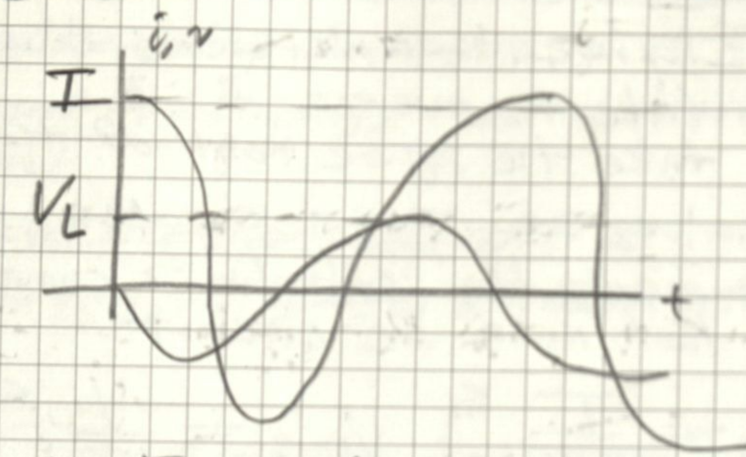
$v_R = IR\cos\omega t$
$= V_R\cos\omega t$

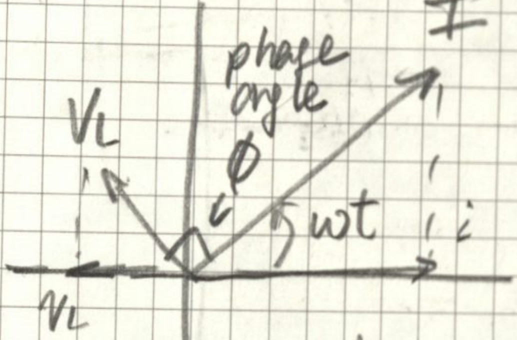


current and voltage **in phase**
i.e. rotate together

Inductor in an ac Circuit:







$i = I\cos\omega t$

$v_L = I\omega L\cos(\omega t + 90°)$

voltage curve/phasor **leads** current phasor by $\phi = \frac{\pi}{2} = 90°$

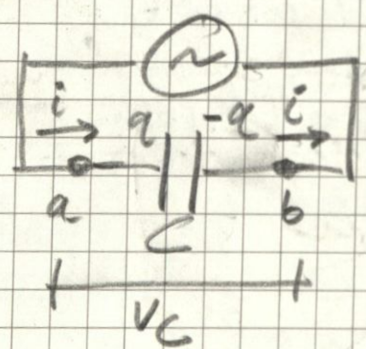
* $v_L = L\frac{di}{dt} = L\frac{d}{dt}(I\cos\omega t) = -I\omega L\sin\omega t = I\omega L\cos(\omega t + 90°)$

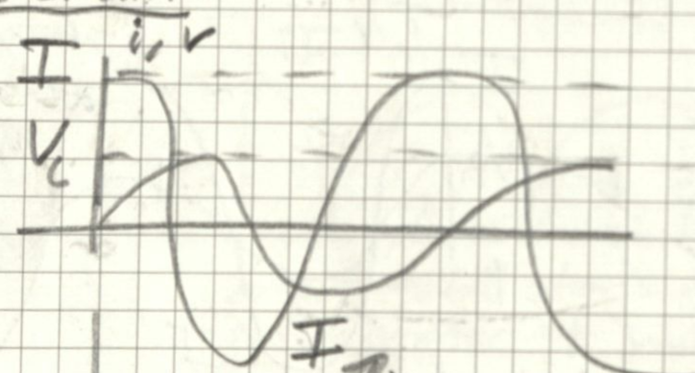
Inductive Reactance: measure of opposition an inductor will have on an AC circuit

$X_L = \omega L$, $V_L = IX_L$

amplitude of voltage across inductor

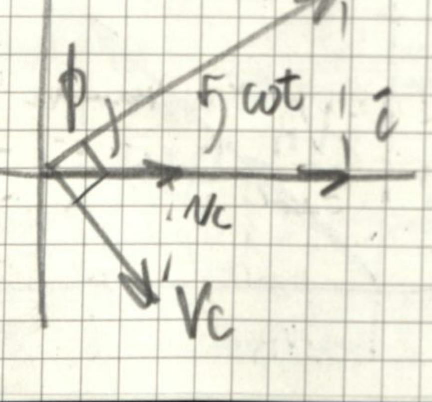
Capacitor in an ac Circuit:





voltage phasor lags current phasor by

$\phi = -\frac{\pi}{2} = -90°$



$i = I\cos\omega t$

$v_C = \frac{1}{\omega C}\cos(\omega t - 90°)$

<u>Derivation</u>:

$i = \frac{dq}{dt} = I\cos\omega t \xrightarrow{\text{integrate}} q = \frac{I}{\omega}\sin\omega t$

$q = Cv_C \Rightarrow v_C = \frac{I}{\omega C}\sin\omega t = \frac{I}{\omega C}\cos(\omega t - 90°$

definition of capacitance

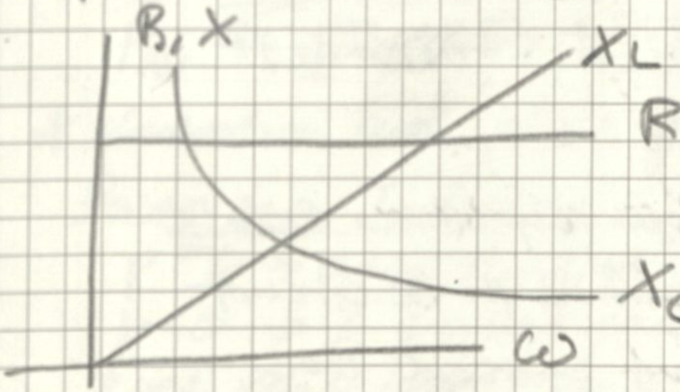
trig identity

<u>Capacitive Reactance</u>: opposition of change in voltage in an AC circuit

$X_C = \frac{1}{\omega C}$, $V_C = IX_C$

<u>Comparing AC Circuit Elements</u>:

Circuit Element	Amplitude Relationship	Circuit Quantity	Phase of v
Resistor	$V_R = IR$	R	in phase with i
Inductor	$V_L = IX_L$	$X_L = \omega L$	leads i by 90°
Capacitor	$V_C = IX_C$	$X_C = \frac{1}{\omega C}$	lags i by 90°



- R independent of ω
- X_C, X_L dependent on ω

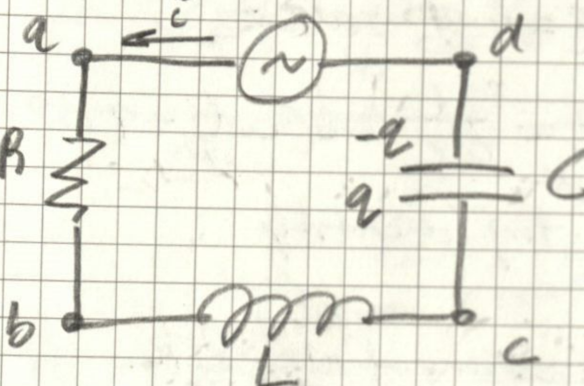
<u>Note</u>: If $\omega = 0$, circuit is effectively DC, no current through capacitor since $X_C \to \infty$, and no inductive effect since $X_L = 0$

If $\omega = \infty$, current through inductor $\to 0$ since $X_L \to \infty$, $X_C \to 0$ so no charge can build on either plate

Reading Notes: 12.2.24

31.3. The L-R-C Series Circuit

Same circuit as in 30.6 except now has AC emf source

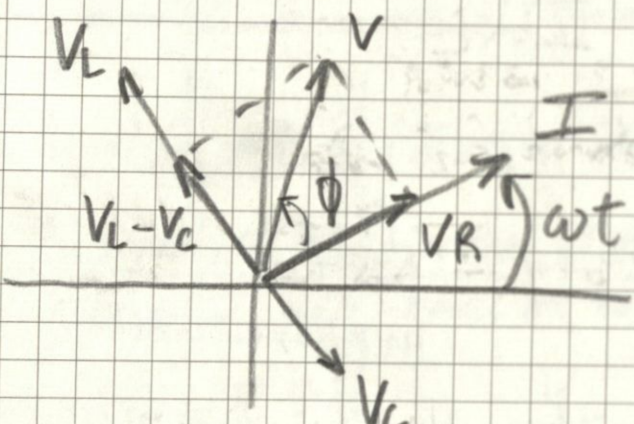


Apply Loop Rule:

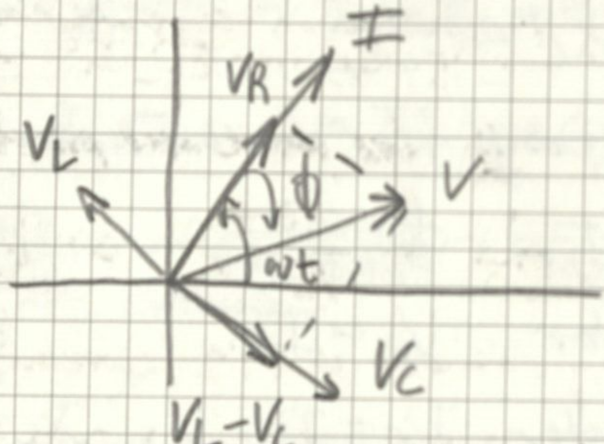
$$L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{q}{C} = V\sin\omega t$$

$\Rightarrow i(t) =$

a) Case $X_L > X_C$



b) Case $X_L < X_C$



V: source voltage phasor (vector sum of V_R, V_L, V_C)

$$V = \sqrt{V_R^2 + (V_L - V_C)^2} = \sqrt{(IR)^2 + (IX_L - IX_C)^2}$$

$$V = I\sqrt{R^2 + (X_L - X_C)^2}$$

Impedance: ratio of voltage amplitude across the circuit to current amplitude in the circuit

i.e. opposition to AC current passed via the combined effect of R, X_L, and X_C

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$ SI: ohm

$$V = IZ$$

Interpreting Impedance and Phase Angle:

* DC circuits → R, $V = IR$
AC circuits → Z, $V = IZ$ *

i.e. just like DC takes path of least resistance, AC takes path of least impedance

Note: Z is a function of $R, L, C,$ and ω

Phase Angle: angle by which the source voltage leads the current

$$\tan\phi = \frac{V_L - V_C}{V_R} = \frac{I(X_L - X_C)}{IR} = \frac{X_L - X_C}{R}$$

Note: we have been discussing these values in terms of maximum amplitudes, but they are typically described in rms values
- all still valid and proportional by $\frac{1}{\sqrt{2}}$

Ex: $\frac{V}{\sqrt{2}} = \frac{I}{\sqrt{2}} Z \Rightarrow V_{RMS} = I_{RMS} Z$

31.4: Power in AC Circuits

instantaneous power to a circuit element: $p = vi$

Power in a Resistor:

$$P_{av} = \frac{1}{2} VI = V_{rms} I_{rms} = I_{rms}^2 R$$

* derivation and phase graphs in textbook *

Power in an Inductor:

$p > 0$ → energy supplied is building a $\vec{B}$

$p < 0$ → $\vec{B}$ is collapsing and energy is returning to source

* net power over 1 cycle is 0 *

Power in a Capacitor: same process as for an inductor

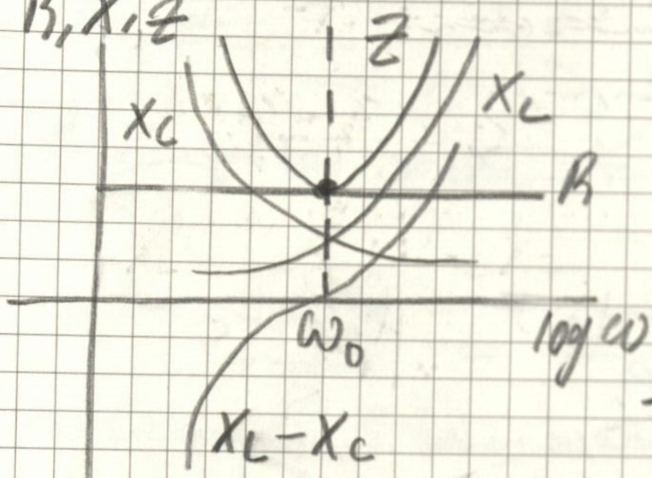
Power in General:

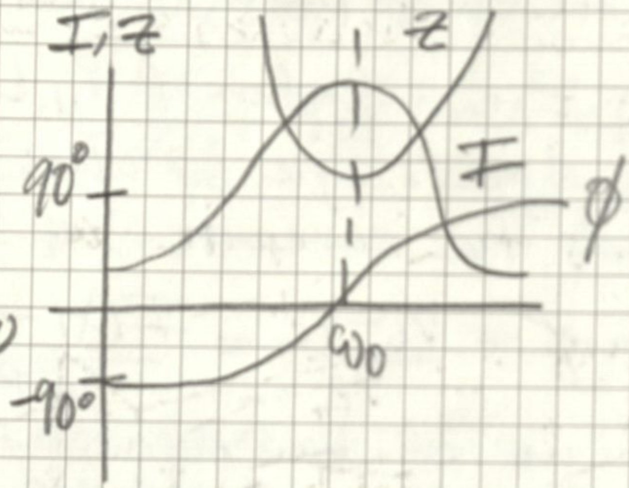
$$P_{av} = \frac{1}{2} VI \cos\phi = V_{rms} I_{rms} \cos\phi$$

Reading Notes :   12.3.24

31.5. Resonance in AC-Circuits

a specific tuning of an LRC series circuit where $X_L = X_C$, hence Z is minimized and equals R





Behavior at Resonance :

Resonance frequency allows for maximum I at a minimum Z

Resonant Angular Frequency : the $\omega = \omega_0$ where this resonance peak occurs

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

Resonance frequency : " . . . " : $f_0 = \frac{\omega_0}{2\pi}$

frequency at which circuit is "tuned"

* at this f_0, $X_L = X_C \Leftrightarrow V_L = V_C$ so total voltage across capacitor and inductor is 0 *

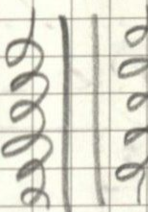
* useful to optimize these circuits — maximum I (output) for minimum V (cost) *

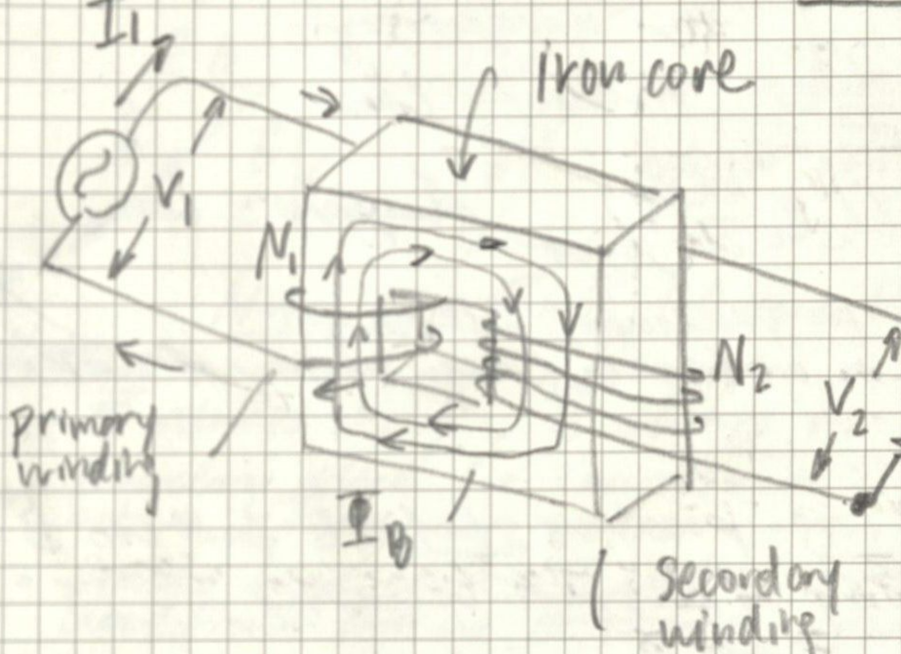
31.6: Transformers

much easier to step voltage levels up and down on an AC circuit using transformers, making them desirable

ex: want high V and low I to transfer power, but low V and high I when using it

Circuit Symbol:





windings: electrically insulated from each other, but wound on the same core

core: keeps $\vec{B}$ induced by coils within the core

maximizes mutual inductance

$$\mathcal{E}_1 = -N_1 \frac{d\Phi_B}{dt} \text{ and } \mathcal{E}_2 = -N_2 \frac{d\Phi_B}{dt}$$

$$\rightarrow \frac{\mathcal{E}_2}{\mathcal{E}_1} = \frac{N_2}{N_1} \Rightarrow \frac{V_2}{V_1} = \frac{N_2}{N_1}$$

Energy in Transformers:

$$V_1 I_1 = V_2 I_2$$

$$\frac{V_1}{I_1} = \frac{R}{(N_2/N_1)^2}$$

derivations in textbook

Reading for 12/6: 12.9.24

32.1 - Maxwell's Equations and Electromagnetic Waves

Electricity, Magnetism, and Light:

$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{encl}}{\varepsilon_0}$ Gauss's Law

$\oint \vec{B} \cdot d\vec{A} = 0$ Gauss's Law for magnetism

$\oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt}$ Faraday's Law

$\oint \vec{B} \cdot d\vec{l} = \mu_0 \left(i_C \varepsilon_0 \frac{d\Phi_E}{dt}\right)_{encl}$ Ampere's Law

* ε_0 and μ_0 used in vacuum, whereas ε and μ used in materials (permeability of material)

Interpretation:

- point charge at rest produces static $\vec{E}$ but no $\vec{B}$
- point charge moving w/ constant velocity produces both $\vec{E}$ and $\vec{B}$
- accelerating point charges produce electromagnetic waves and radiate electromagnetic energy

Electromagnetic Radiation

- used interchangeably w/ "electromagnetic waves"
- point charges in SHM have an acceleration at all instances (except equilibrium) and emit electromagnetic waves

* drawings of these field lines in textbook

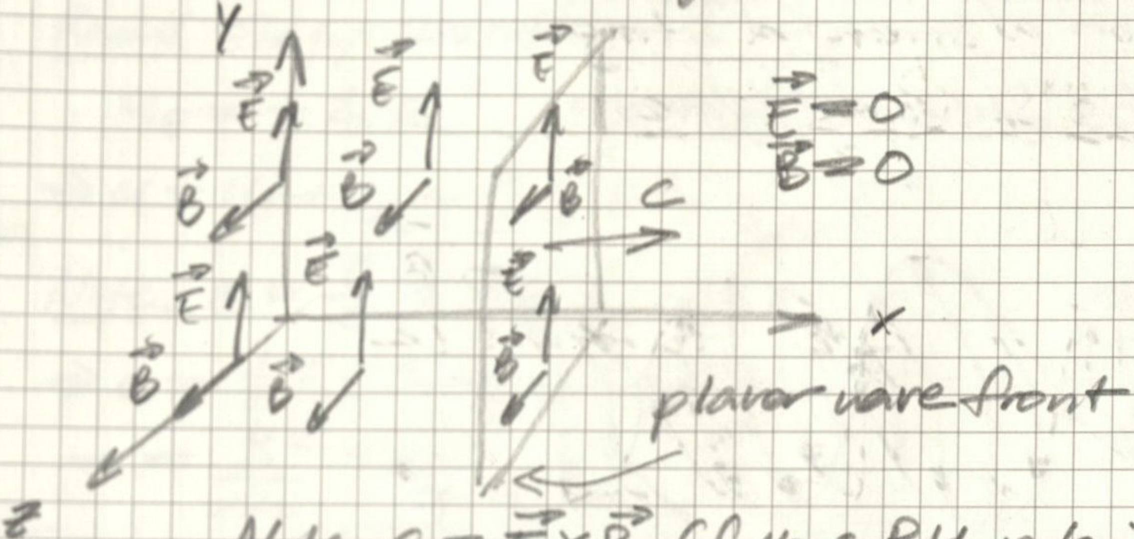
Wavelength-Frequency Relationship: $v = \lambda f$

- for electromagnetic waves in a vacuum:

$c = \lambda f$ ...

32.2 : Plane Electromagnetic Waves and the Speed of Light

A Simple Plane Electromagnetic Wave :



Note. $c = \vec{E} \times \vec{B}$ (follows RH rule)

Note :

- $\vec{E}$ always in $\hat{j}$, $\vec{B}$ always in $\hat{k}$
- Boundary plane (ie "wavefront") moves in $\hat{i}$ at a speed c
- fields have constant values to the left of wavefront and are 0 to the right

* these properties make electromagnetic waves hold for all of Maxwell's Equations *

→ derivations / justifications in book

from Gauss's laws: wave must be transverse

ie. $\vec{E} \perp \vec{B} \perp c$

from Faraday's law: $E = cB$

from Ampere's law: $B = \varepsilon_0 \mu_0 c E$

Using All Eqns : $c = \dfrac{1}{\sqrt{\varepsilon_0 \mu_0}}$

* derivation of Electromagnetic Wave Equation also in textbook *

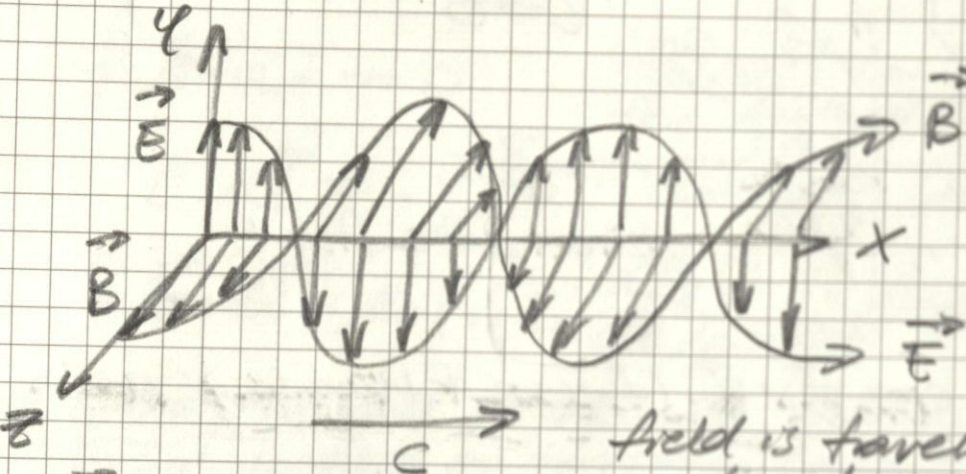
$$\frac{\partial^2 E}{\partial x^2} = \mu_0 \varepsilon_0 \frac{\partial^2 E}{\partial t^2}$$ (in vacuum)

Reading Notes: 12/9/24

32.3. Sinusoidal Electromagnetic Waves

* directly analogous to sinusoidal mechanical waves on a stretched string

Fields of a Sinusoidal Wave:



field is traveling in $\vec{E} \times \vec{B}$ direction

$\vec{E}$: y-component only
$\vec{B}$: z-component only

CAUTION: $\vec{E}$ and $\vec{B}$ are everywhere, not only on the y and z-axis

Wave Functions:

wave number

$$\vec{E}(x,t) = \hat{j}\, E_{max} \cos(kt - \omega t)$$

$$\vec{B}(x,t) = \hat{k}\, B_{max} \cos(kt - \omega t)$$

$$E_{max} = c B_{max}$$

* sinusoidal waves can also exist with $\vec{E}$ or $\vec{B}$ propagating in opposite directions! *

* more about electromagnetic waves in matter and permeability in the textbook *

Reading Notes: 12.10.24

§32.4. Energy and Momentum in Electromagnetic Waves

Recall. Energy Densities

$$u_E = \frac{1}{2}\varepsilon_0 E^2 , \quad u_B = \frac{B^2}{2\mu_0}$$

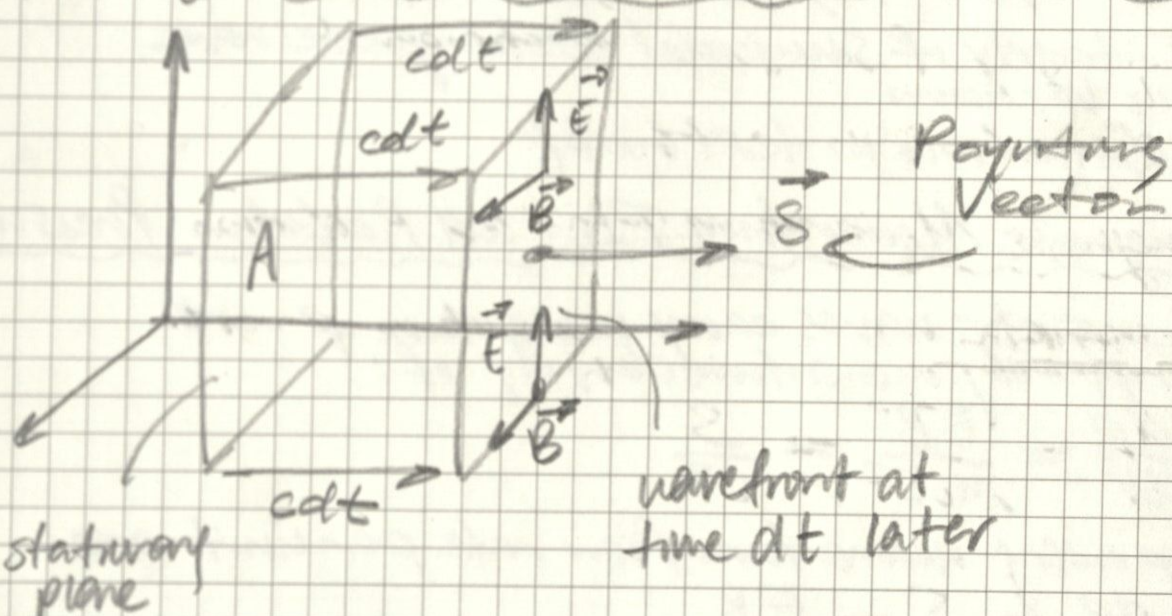
$$\Rightarrow u = u_E + u_B = \frac{1}{2}\varepsilon_0 E^2 + \frac{1}{2\mu_0} B^2$$

Note. $B = \frac{E}{c} = \sqrt{\varepsilon_0 \mu_0}\, E$

$$\Rightarrow u = \frac{1}{2}\varepsilon_0 E^2 + \frac{1}{2\mu_0}\left(\sqrt{\varepsilon_0\mu_0}\, E\right)^2 = \varepsilon_0 E^2$$

$$\ldots = \frac{B^2}{\mu_0}$$

Electromagnetic Energy Flow and the Poynting Vector:



$$dU = u\,dV = (\varepsilon_0 E^2)(A c\, dt)$$

$$S = \frac{1}{A}\frac{dU}{dt} = \varepsilon_0 c E^2 \quad \text{(in vacuum)}$$

↳ energy flow per unit time per unit area

$$= \ldots = \frac{\varepsilon_0}{\sqrt{\varepsilon_0\mu_0}} E^2 = \sqrt{\frac{\varepsilon_0}{\mu_0}}\, E^2 = \frac{EB}{\mu_0}$$

* all useful derivations on formula sheet

SI. $1\ \mathrm{J/s\cdot m^2} = 1\ \mathrm{W/m^2}$

Poynting Vector: magnitude and direction of the energy flow rate

$$\vec{S} = \frac{1}{\mu_0} \vec{E} \times \vec{B}$$

↳ Poynting vector in vacuum
(points in direction of propagation of wave)

$$P = \oint \vec{S} \cdot d\vec{A}$$

↳ power

Intensity: → magnitude of average Poynting vector

$$I = S_{av} = \frac{E_{max} B_{max}}{2\mu_0} = \frac{E_{max}^2}{2\mu_0 c} = \frac{1}{2}\sqrt{\frac{\varepsilon_0}{\mu_0}}\, E_{max}^2 = \frac{1}{2}\varepsilon_0 c E_{max}^2$$

↳ Intensity of sinusoidal electromagnetic wave in vacuum

& derivation in textbook

Electromagnetic Momentum Flow and Radiation Pressure:

electromagnetic waves carry momentum p with a corresponding momentum density of:

$$\frac{dp}{dV} = \frac{EB}{\mu_0 c^2} = \frac{S}{c^2}$$

corresponding momentum flow rate per unit area:

$$\frac{1}{A}\frac{dp}{dt} = \frac{S}{c} = \frac{EB}{\mu_0 c}$$

can obtain average rate of momentum transfer by replacing S with $S_{av} = I$ which is called radiation pressure:

$$P_{rad} = \frac{S_{av}}{c} = \frac{I}{c} \quad \text{(wave totally absorbed)}$$

$$P_{rad} = \frac{2S_{av}}{c} = \frac{2I}{c} \quad \text{(wave totally reflected)}$$