Engineering Physics I: Reading for Class #3:   1.18.24

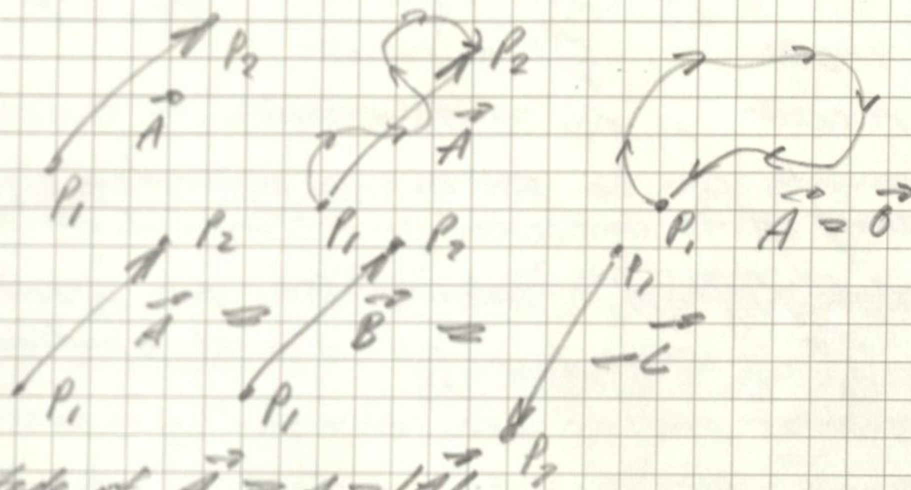
1.6: Estimates and Orders of Magnitude: special kind of estimation

ex- Gold sells for \$1400 an ounce
or \$100 for $\frac{1}{14}$ of an ounce

1.7: Vectors and vector addition

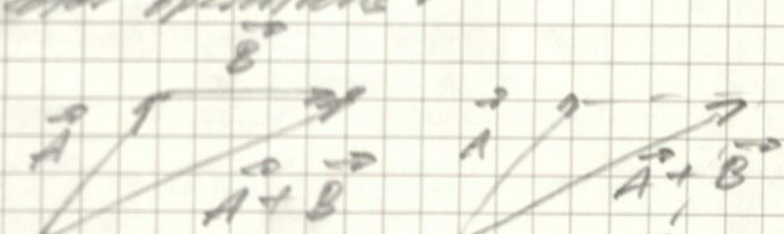
scalars → quantity described by a single number
vectors → quantity described by multiple numbers
∘ magnitude and direction

* displacement is a vector quantity *
∘ it has both magnitude and direction



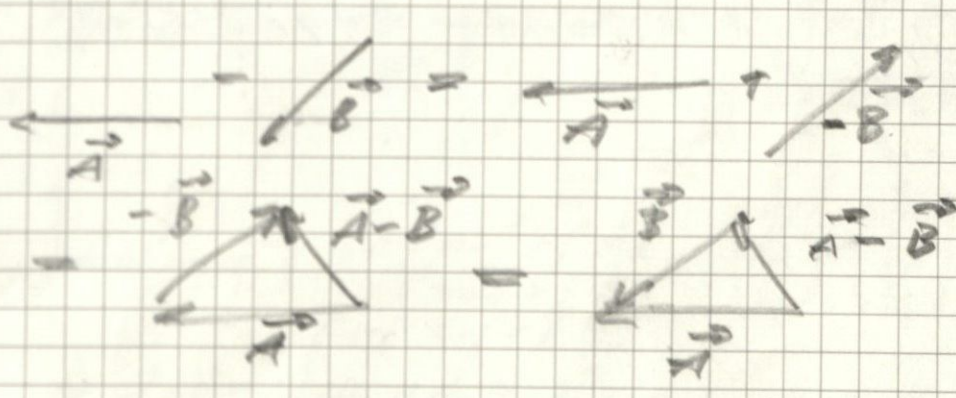
magnitude of $\vec{A} = A = |\vec{A}|$
* always positive *

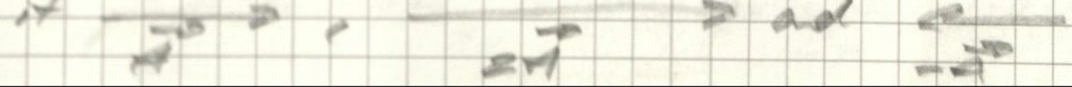
Vector operations:



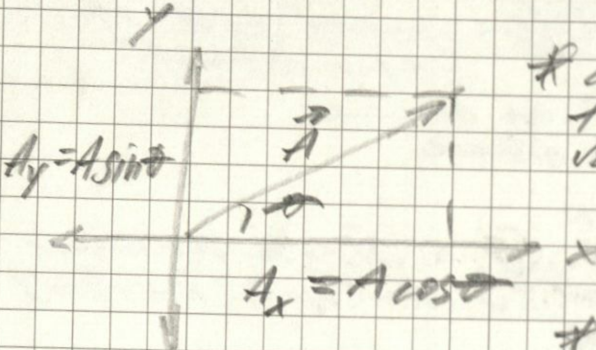
$|\vec{A}+\vec{B}| \neq |\vec{A}|+|\vec{B}|$
unless $\vec{A} \parallel \vec{B}$

$\vec{A}-\vec{B} = \vec{A}+(-\vec{B})$:





1.8: Components of Vectors:



* components of $\vec{A}$ are the projections of the vector on the x and y axis

* in this case, both A_x and A_y are positive

* components are NOT vectors, they are scalars

θ — distance above horizontal w/ "angle from +x-axis to +y-axis"?

$\frac{A_x}{A} = \cos\theta$, $\frac{A_y}{A} = \sin\theta$

$A_x = A\cos\theta$, $A_y = A\sin\theta$

* components can be pos or neg depending on the quadrant of the cartesian grid they reside in

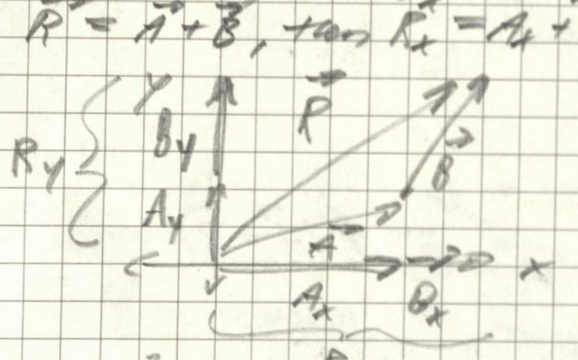
$|\vec{A}| = A = \sqrt{A_x^2 + A_y^2}$ (always take pos square-root)

$\tan\theta = \frac{A_y}{A_x}$, $\theta = \tan^{-1}\frac{A_y}{A_x}$ * check quadrant *

* always draw pictures b/c $\theta = \tan^{-1}(+1) = 45° = 225°$

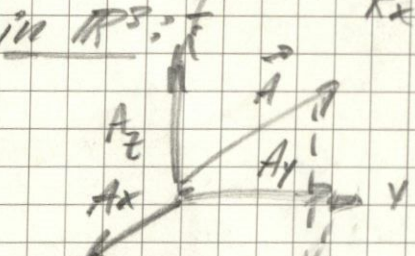
if $\vec{D} = c\vec{A}$, then $D_x = cA_x$ and $D_y = cA_y$

if $\vec{R} = \vec{A} + \vec{B}$, then $R_x = A_x + B_x$ and $R_y = A_y + B_y$



* applies to any sum of n vectors *

in $\mathbb{R}^3$:

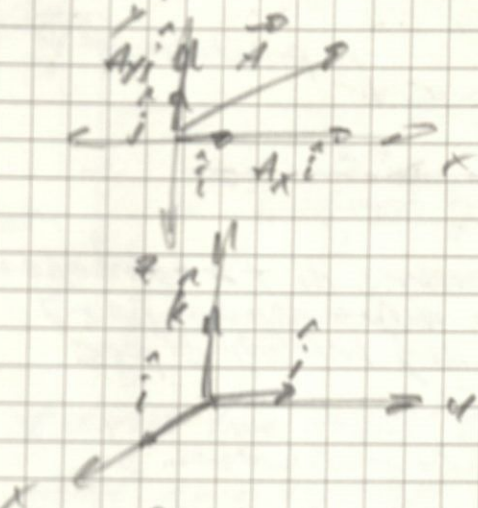


* works in any n-dimensions

* in this case $A = \sqrt{A_x^2 + A_y^2 + A_z^2}$

1.9: Unit Vectors

* a vector with a magnitude of 1, and no units
  its purpose is to point



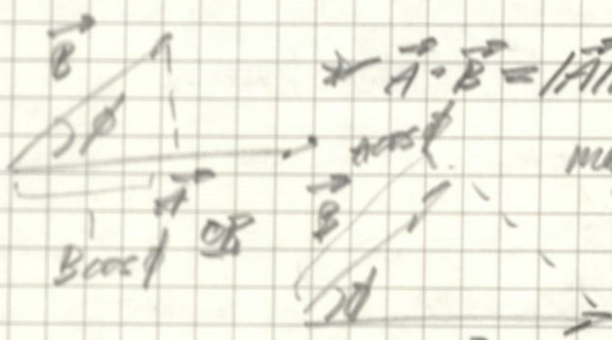
$$\vec{A} = A_x\hat{i} + A_y\hat{j}$$

$$\vec{R} = \vec{A} + \vec{B}$$
$$= (A_x\hat{i} + A_y\hat{j}) + (B_x\hat{i} + B_y\hat{j})$$
$$= (A_x + B_x)\hat{i} + (B_x + B_y)\hat{j}$$
$$= R_x\hat{i} + R_y\hat{j}$$

$$\Rightarrow \vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$$

1.10: Products of Vectors

Scalar Product "dot product":



* $\vec{A}\cdot\vec{B} = |\vec{A}||\vec{B}|\cos\phi = \underbrace{A}_{\text{magnitude of } \vec{A}}\underbrace{B\cos\phi}_{\text{component of } \vec{B} \text{ in direction of } \vec{A}}$

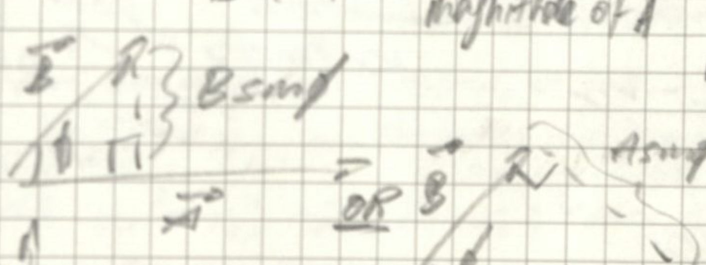
$\vec{A}\cdot\vec{B} = \underbrace{B}_{\text{magnitude of } \vec{B}}(\underbrace{A\cos\phi}_{\text{component of } \vec{A} \text{ in direction of } \vec{B}})$

* $\vec{A}\cdot\vec{B} = A_xB_x + A_yB_y + A_zB_z \rightsquigarrow \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} \cdot \begin{bmatrix} B_x \\ B_y \\ B_z \end{bmatrix}$

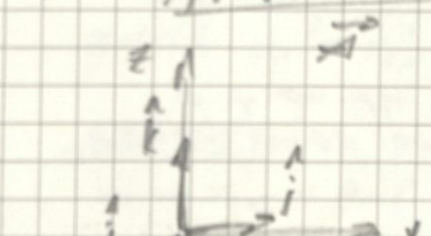
Vector Product "cross product":

* $|\vec{C}| = |\vec{A}||\vec{B}|\sin\phi \Rightarrow C = \underbrace{A}_{\text{magnitude of } \vec{A}}\underbrace{B\sin\phi}_{\text{component of } \vec{B} \text{ perpendicular to } \vec{A}}$

$\vec{A}\times\vec{B} = -\vec{B}\times\vec{A}$



$\vec{A}\times\vec{B} = \underbrace{B}_{\text{magnitude of } \vec{B}}(\underbrace{A\sin\phi}_{\text{comp of } \vec{A} \text{ perpendicular to } \vec{B}})$



$\hat{i}\times\hat{j} = -\hat{j}\times\hat{i} = \hat{k}$
$\hat{j}\times\hat{k} = -\hat{k}\times\hat{j} = \hat{i}$
$\hat{k}\times\hat{i} = -\hat{i}\times\hat{k} = \hat{j}$

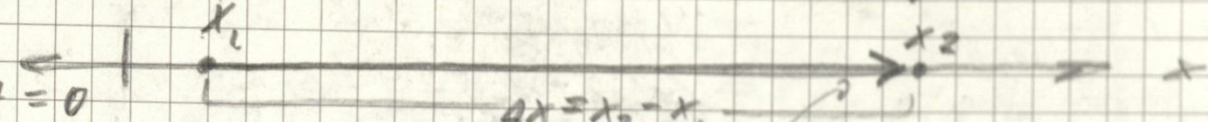
$$\begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} \times \begin{bmatrix} B_x \\ B_y \\ B_z \end{bmatrix} = \begin{bmatrix} A_yB_z - A_zB_y \\ A_zB_x - A_xB_z \\ A_xB_y - A_yB_x \end{bmatrix}$$

* $\vec{A}\times\vec{B} = \underbrace{(A_yB_z - A_zB_y)}_{C_x}\hat{i} + \underbrace{(A_zB_x - A_xB_z)}_{C_y}\hat{j} + \underbrace{(A_xB_y - A_yB_x)}_{C_z}\hat{k}$

Reading for Class #3: 1/21/24

2.1: Displacement, Time, and Average Velocity

x-graphs: position at t_1 position at t_2



$\Delta x = x_2 - x_1$

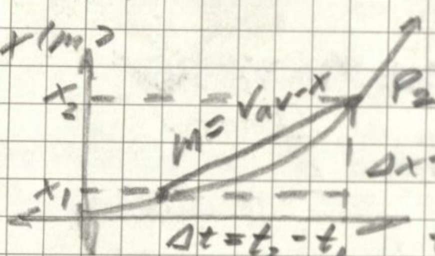
$\Delta t = t_2 - t_1$

moving in $+x$-direction, so displacement (Δx) and avg velocity is pos

$v_{av\text{-}x} = \frac{\Delta x}{\Delta t} = \dots$ m/s

rules for sign of v_x:
- pos and inc → pos v_x ← ($+x$ direction)
- pos and dec → neg v_x ← ($-x$ direction)
- neg and inc → pos v_x
- neg and dec → neg v_x * same for a_x *

$x(t)$-graphs



2.2: Instantaneous Velocity

$v_x = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$ $\quad v = |\vec{v}_x| \quad v \neq |\vec{v}_{av\text{-}x}|$

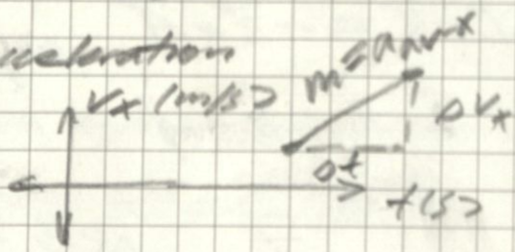
v: symbol for speed (how fast)

v_x: symbol for velocity (how fast and in what direction)

2.3: Average and Instantaneous Acceleration

$a_{av\text{-}x} = \frac{\Delta v_x}{\Delta t} = \frac{v_{2x} - v_{1x}}{t_2 - t_1}$

$= \dots$ m/s^2



$a_x = \lim_{\Delta t \to 0} \frac{\Delta v_x}{\Delta t} = \frac{dv_x}{dt}$

concave up → $+a_x$

concave down → $-a_x$

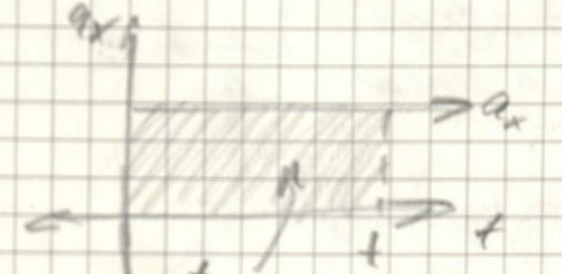
$a_x = \frac{dv_x}{dt} = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d^2x}{dt^2}$

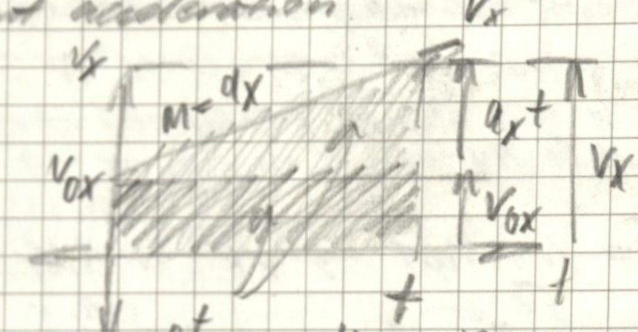
	on x-t graph	on v_x-t graph
value of graph	coordinate x	velocity v_x
slope of graph	velocity v_x	acceleration a_x
concavity of graph	acceleration a_x	Δa_x

* at a particular time $t = t^*$

Reading for Class #4: 1.23.24

2.4: Motion with Constant acceleration





$\int_0^t a_x(t)\,dt = v_x - v_{0x}$ $\int_0^t v_x(t)\,dt = x - x_0$

during time interval t, v_x changes by $v_x - v_{0x} = a_x t$

when a_x is constant, $a_{av\text{-}x} = a_x$ at any t:

so $a_x = \frac{v_{2x} - v_{1x}}{t_2 - t_1} \xrightarrow{t_1 = 0} \frac{v_x - v_{0x}}{t - 0}$ "v_{0x}" → initial velocity

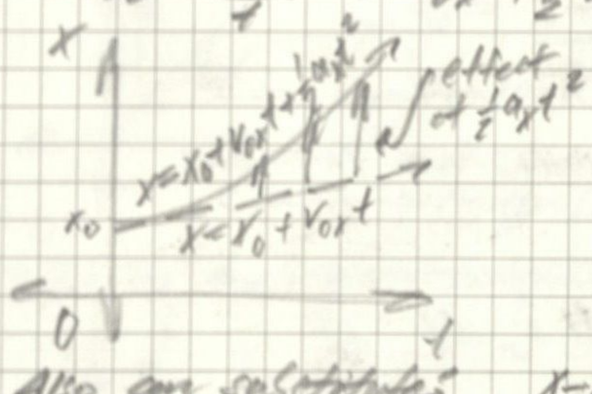
or $v_x = v_{0x} + a_x t$. Also: "x_0" → initial position

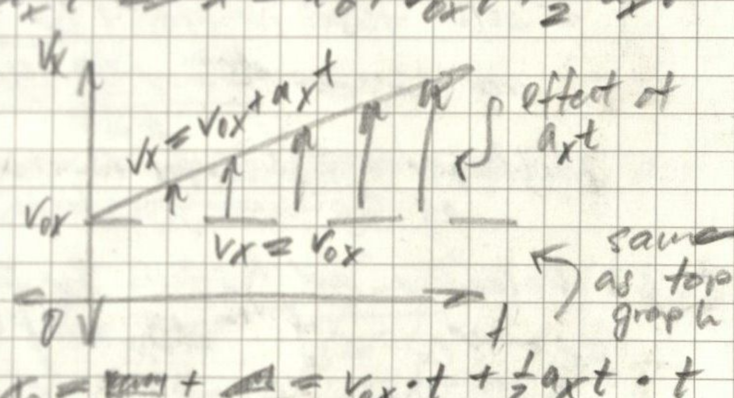
$v_{av\text{-}x} = \frac{x - x_0}{t}$ where $t = 0 \Rightarrow \Delta t = t - 0 \Leftrightarrow \Delta x = x - x_0$

and $v_{av\text{-}x} = \frac{1}{2}(v_{0x} + v_x)$, so:

$v_{av\text{-}x} = \frac{1}{2}(v_{0x} + v_{0x} + a_x t) = v_{0x} + \frac{1}{2} a_x t$,

or $\frac{x - x_0}{t} = v_{0x} + \frac{1}{2} a_x t \Rightarrow x = x_0 + v_{0x} t + \frac{1}{2} a_x t^2$





Also can substitute: $x - x_0 = \cancel{\quad} + \cancel{\quad} = v_{0x} \cdot t + \frac{1}{2} a_x t \cdot t$

$t = \frac{v_x - v_{0x}}{a_x}$, (equating two expressions for $v_{av\text{-}x}$ from above and multiplying by t)

giving $v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$ or $x - x_0 = \frac{1}{2}(v_{0x} + v_x)t$

<u>equation:</u>	<u>relates:</u>			
$v_x = v_{0x} + a_x t$	t		v_x	a_x
$x = x_0 + v_{0x} t + \frac{1}{2} a_x t^2$	t	x		a_x
$v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$		x	v_x	a_x
$x - x_0 = \frac{1}{2}(v_{0x} + v_x)t$	t	x	v_x	

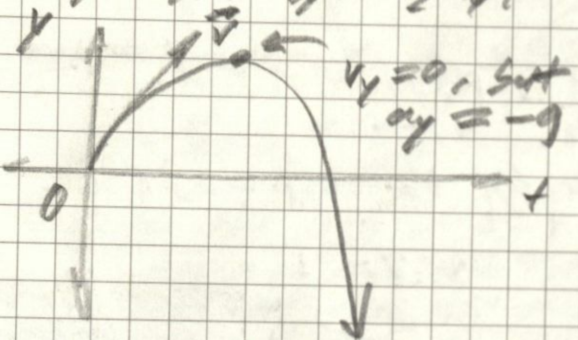
2.5: Freely Falling Objects

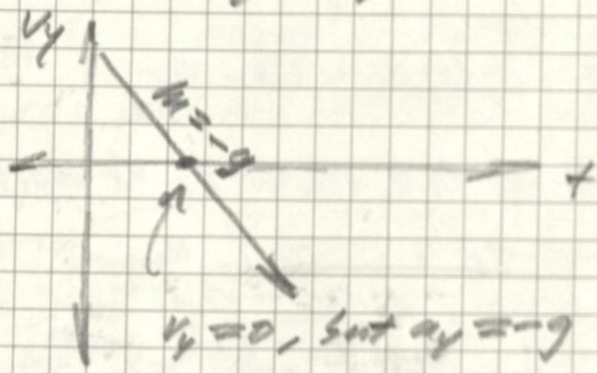
gravity (g) is a downward acceleration force that is constant and independent of weight

$$g = 9.8\,m/s^2 = 980\,cm/s^2 = 32.2\,ft/s^2$$

* g is always positive, as it is a scalar
* if +y-axis is up, $a_y = -g$

$$y = y_0 + v_{0y}t + \frac{1}{2}a_y t^2 \qquad v_y = v_{0y} + a_y t$$





CAUTION: don't confuse speed, velocity, and acceleration
- speed can never be negative
- velocity can be positive or negative
- acceleration is constant and downward

both change continuously

* if a freely falling object passes a given point at two different times (once moving upward and once moving downward), its <u>speed</u> will be the same at both times

* solve time in $Ax^2 + Bx + C = 0$ and $x = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$ often
(negative solutions for x (t) are fictitious as you cannot have negative time)

2.6: Velocity and Position by Integration

$$v_{2x} - v_{1x} = \int_{v_{1x}}^{v_{2x}} dv_x = \int_{t_1}^{t_2} a_x\,dt$$

$$x_2 - x_1 = \int_{x_1}^{x_2} dx = \int_{t_1}^{t_2} v_x\,dt$$

$$\left.\begin{aligned} v_x &= v_{0x} + \int_0^t a_x\,dt \\ x &= x_0 + \int_0^t v_x\,dt \end{aligned}\right\}$$

for motion with changing acceleration and velocity

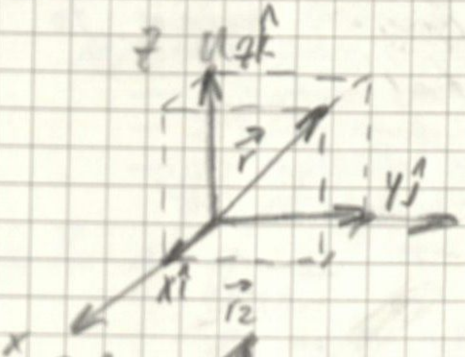
ALTERNATIVELY: if a is constant

$$v = \int a\,dt = a\int dt = at + v_0 \quad (+C)$$

$$x = \int v\,dt = \int (at + v_0)\,dt = \frac{1}{2}at^2 + v_0 t + x_0$$

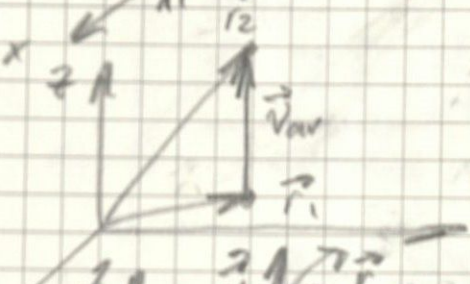
Reading for Class #5: 1.25.24

3.1: Position and Velocity Vectors



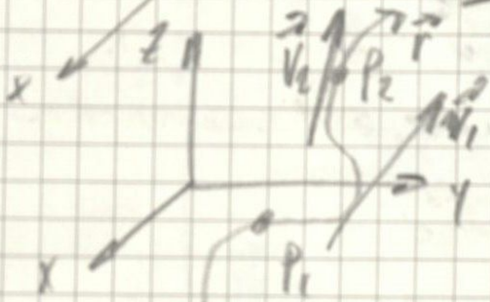
$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$

$$\begin{bmatrix} r_x \\ r_y \\ r_z \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$



$$\vec{v}_{av} = \frac{\Delta \vec{r}}{\Delta t} = \frac{\vec{r}_2 - \vec{r}_1}{t_2 - t_1}$$

$$\begin{bmatrix} v_{av-x} \\ v_{av-y} \\ v_{av-z} \end{bmatrix} = \begin{bmatrix} \Delta r_x / \Delta t \\ \Delta r_y / \Delta t \\ \Delta r_z / \Delta t \end{bmatrix}$$



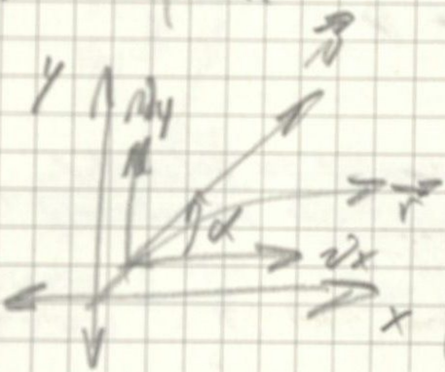
$$\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta \vec{r}}{\Delta t} = \frac{d\vec{r}}{dt} \rightsquigarrow \begin{bmatrix} v_x \\ v_y \\ v_z \end{bmatrix} = \begin{bmatrix} dr_x/dt \\ dr_y/dt \\ dr_z/dt \end{bmatrix}$$

"tangent to the path at each point"

$$\vec{v} = \frac{d\vec{r}}{dt} = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j} + \frac{dz}{dt}\hat{k}$$

$$|\vec{v}| = v = \sqrt{v_x^2 + v_y^2 + v_z^2} \quad \text{or} \quad \sqrt{v_x^2 + v_y^2}$$

"speed"

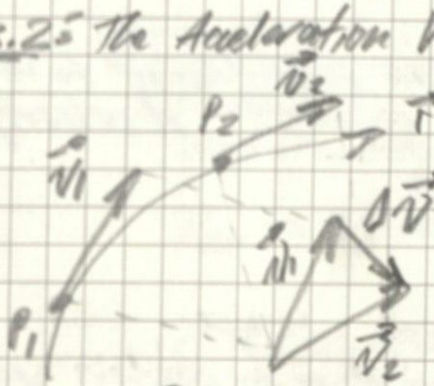


$$\tan\alpha = \frac{v_y}{v_x}$$

"direction of instantaneous velocity, not to be confused with θ"

3.2: The Acceleration Vector

$$\vec{a}_{av} = \frac{\Delta \vec{v}}{\Delta t} = \frac{\vec{v}_2 - \vec{v}_1}{t_2 - t_1}$$

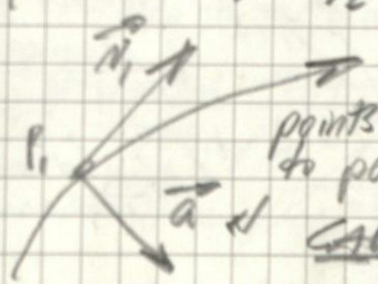


$$\Delta\vec{v} = \vec{v}_2 - \vec{v}_1$$

$$\begin{bmatrix} a_{av-x} \\ a_{av-y} \\ a_{av-z} \end{bmatrix} = \begin{bmatrix} \Delta v_x / \Delta t \\ \Delta v_y / \Delta t \\ \Delta v_z / \Delta t \end{bmatrix}$$

$$\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t} = \frac{d\vec{v}}{dt} \rightsquigarrow \begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix} = \begin{bmatrix} dv_x/dt \\ dv_y/dt \\ dv_z/dt \end{bmatrix}$$

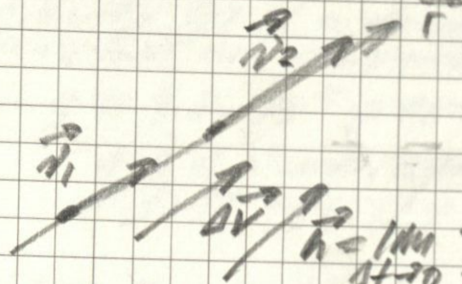
$$= \frac{dv_x}{dt}\hat{i} + \frac{dv_y}{dt}\hat{j} + \frac{dv_z}{dt}\hat{k}$$



points concave to path

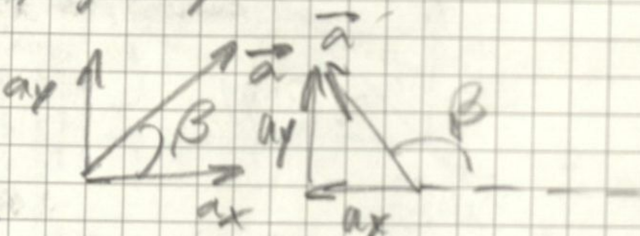
$$= \frac{d^2x}{dt^2}\hat{i} + \frac{d^2y}{dt}\hat{j} + \frac{d^2z}{dt}\hat{k}$$

CAUTION: Any particle following a curved path is accelerating



$$\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t}$$

"only if the trajectory is a straight line is the acceleration tangent to the trajectory"

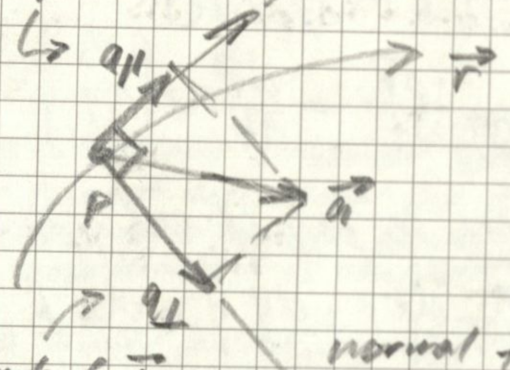


"direction of instantaneous acceleration"

$$\tan\beta = \frac{a_y}{a_x}$$

component of $\vec{a}$ parallel to path

$\vec{v}$ "tangent to path at P"



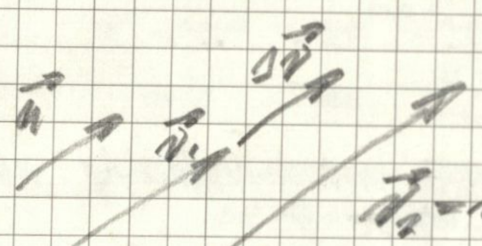
$$a = \sqrt{a_\parallel^2 + a_\perp^2}$$

$a_\parallel = a\cos\beta$

$a_\perp = a\sin\beta$

normal to path at P

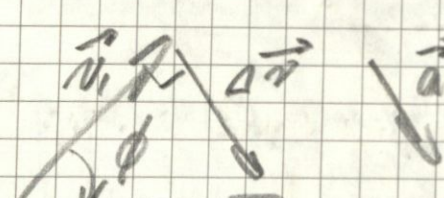
component of $\vec{a}$ perpendicular to path



"acceleration parallel to velocity changes only magnitude of velocity"

$a_\perp = 0$

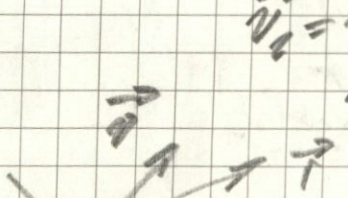
$\vec{v}_2 = \vec{v}_1 + \Delta\vec{v}$



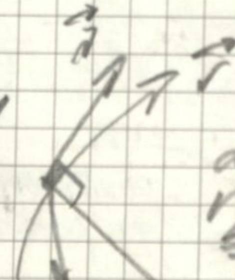
"acceleration perpendicular to velocity changes only direction of velocity"

$a_\parallel = 0$

$\vec{v}_2 = \vec{v}_1 + \Delta\vec{v}$



"when speed is increasing along a curved path, $\vec{a}$ points ahead of the normal"

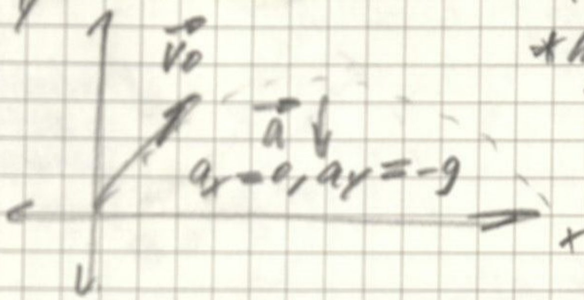


"when speed is decreasing along a curved path, $\vec{a}$ points behind of the normal"

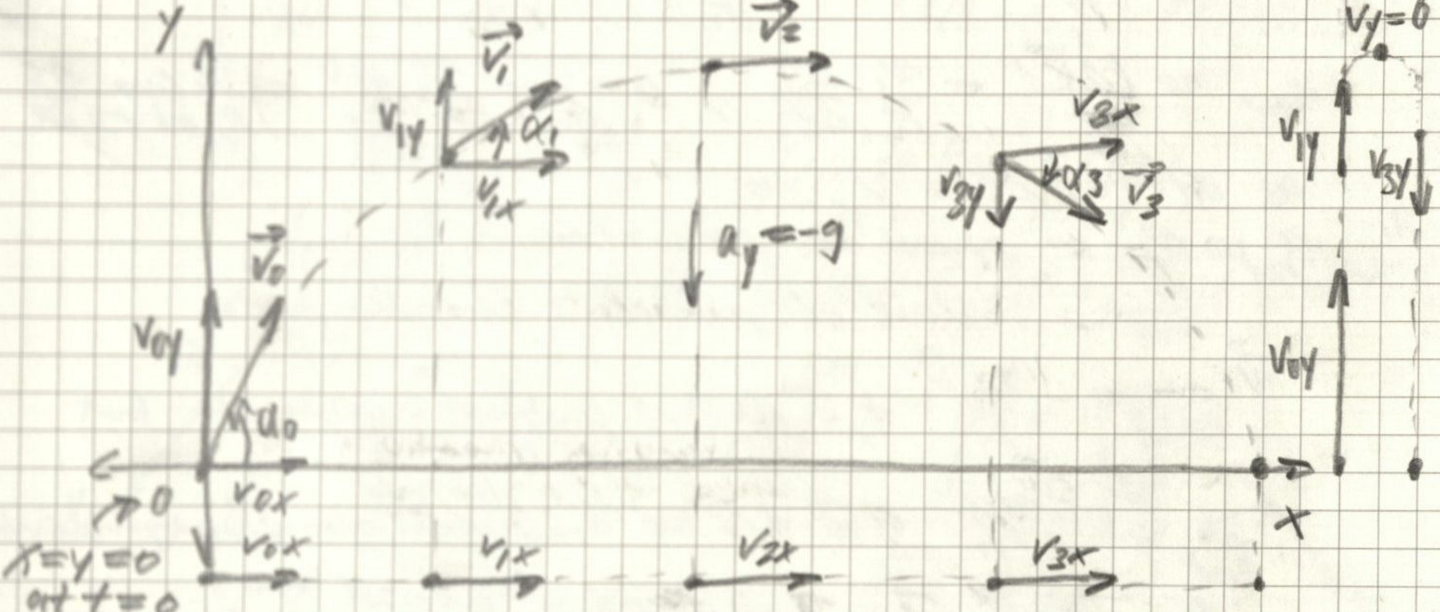
Reading for Class #6: 1.28.24

3.3.5 Projectile Motion

By definition, Projectile Motion:
- Ignores curvature of the Earth
- Ignores air resistance
- Is confined to a 2D-plane in the direction of its initial velocity



* horizontal motion has no effect on vertical motion
* analyze problems as a combination of horizontal motion with constant velocity, and vertical motion with constant acceleration



$v_{0x} = v_{1x} = v_{2x} = v_{3x}$

x-motion:

$v_x = v_{0x}$

$x = x_0 + v_{0x} t$

$v_{0x} = v_0 \cos\alpha_0$

$x = (v_0 \cos\alpha_0) t$

$v_x = v_0 \cos\alpha_0$

y-motion:

$v_y = v_{0y} - gt$

$y = y_0 + v_{0y} t - \frac{1}{2} g t^2$

$v_{0y} = v_0 \sin\alpha_0$

$y = (v_0 \sin\alpha_0) t - \frac{1}{2} g t^2$

$v_y = v_0 \sin\alpha_0 - gt$

General Motion:

$r = \sqrt{x^2 + y^2}$

$v = \sqrt{v_x^2 + v_y^2}$

$\tan\alpha = \frac{v_y}{v_x}$

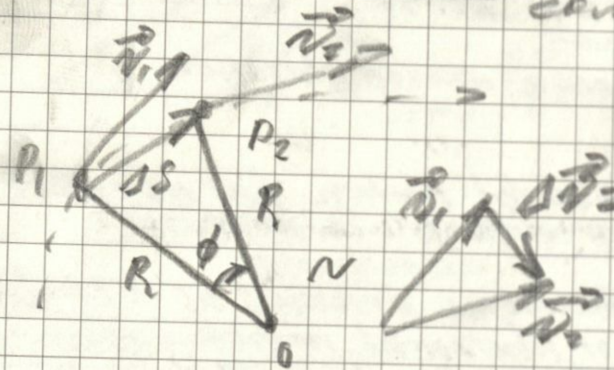
$y = (\tan\alpha_0) x - \frac{g}{2 v_0^2 \cos^2\alpha_0} x^2$

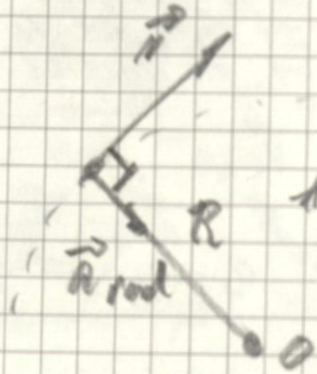
$\approx bx - cx^2$

Reading for Class #8: 2.1.24

3.4: Motion in a Circle

Uniform Circular Motion: particle moving in a circle with constant speed





* always pointing inward

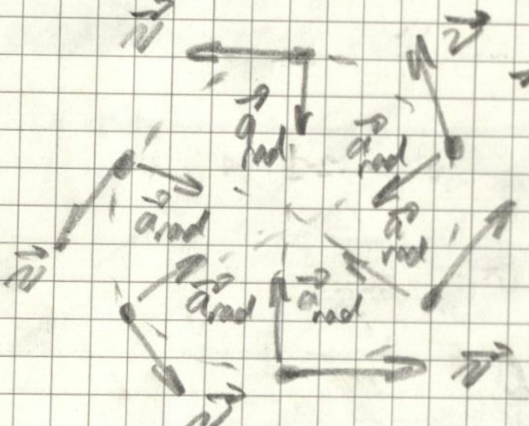
$$\frac{|\Delta \vec v|}{v_1} = \frac{\Delta s}{R} \Rightarrow |\Delta \vec v| = \frac{v_1}{R}\Delta s$$

$$a_{av} = \frac{|\Delta \vec v|}{\Delta t} = \frac{v_1}{R}\frac{\Delta s}{\Delta t}, \quad a = \lim_{\Delta t \to 0} \frac{v_1}{R}\frac{\Delta s}{\Delta t} = \frac{v_1}{R}\lim_{\Delta t \to 0}\frac{\Delta s}{\Delta t}$$

as $\Delta t \to 0$, $\Delta s = v$, so

$a_c \to a_{rad} = \frac{v^2}{R}$ ← speed of particle

← radius of particle's circular path



* $\vec a_{rad}$ has constant magnitude but varying direction

$\vec v$ & $\vec v$ and $\vec a_{rad}$ are always perpendicular

a in terms of period T:

$$v = \frac{2\pi R}{T}, \quad a_{rad} = \frac{4\pi^2 R}{T^2}$$

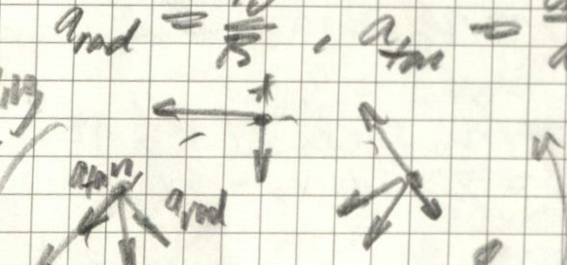
RPM: rotations/revolutions per min.

Nonuniform Circular Motion:

$$a_{rad} = \frac{v^2}{R}, \quad a_{tan} = \frac{d|\vec v|}{dt}$$

* a_{rad} minimum, a_{tan} zero → speed slowest

speeding up, $a_{tan} \sim \vec v$



slowing down, a_{tan} opposite to $\vec v$

a_{rad} maximum, a_{tan} zero → fastest

Note: $\frac{d|\vec v|}{dt} \neq \left|\frac{d\vec v}{dt}\right|$

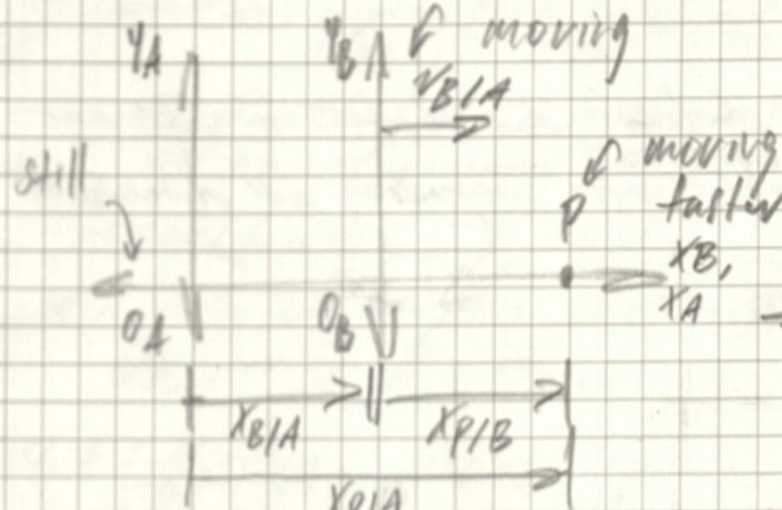
and $\left|\frac{d\vec v}{dt}\right| = \sqrt{a_{rad}^2 + a_{tan}^2}$

$\Rightarrow \vec a$

Reading for Class #7 2.4.24

3.5: Relative Velocity

Frame of Reference: each observer forms a frame of reference and consists of a coordinate system and time scale

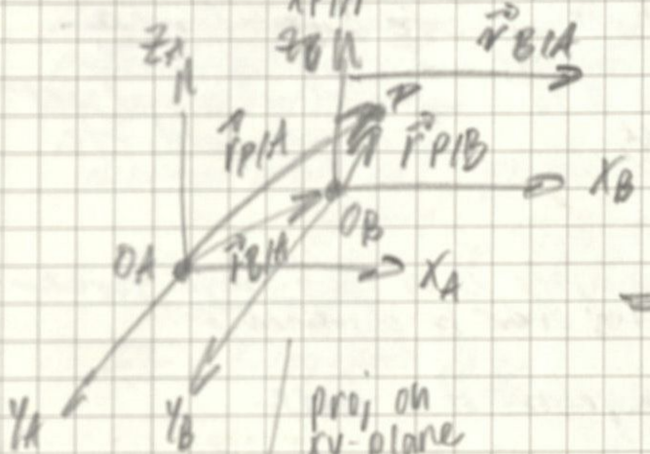


$$x_{P/A} = x_{P/B} + x_{B/A}$$

$$\frac{dx_{P/A}}{dt} = \frac{dx_{P/B}}{dt} + \frac{dx_{B/A}}{dt}$$

$$\Rightarrow v_{P/A-x} = v_{P/B-x} + v_{B/A-x}$$

$$v_{A/B-x} = -v_{B/A-x}$$

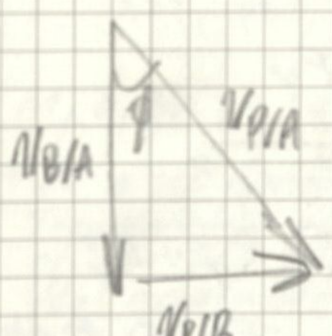


$$\vec{r}_{P/A} = \vec{r}_{P/B} + \vec{r}_{B/A}$$

$$\frac{d\vec{r}_{P/A}}{dt} = \frac{d\vec{r}_{P/B}}{dt} + \frac{d\vec{r}_{B/A}}{dt}$$

$$\Rightarrow \vec{v}_{P/A} = \vec{v}_{P/B} + \vec{v}_{B/A}$$

$$\vec{v}_{A/B} = -\vec{v}_{B/A}$$



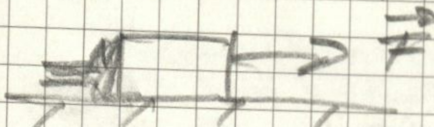
$$\tan\theta = \frac{v_{P/B}}{v_{B/A}}$$

$$v_{P/A} = \sqrt{v_{P/B}^2 + v_{B/A}^2}$$

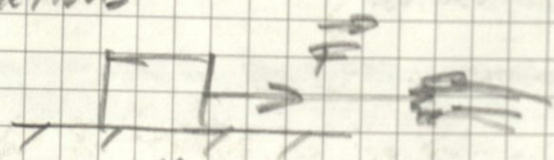
Reading for Class #10:

2.6.24

4.1: Force and Interactions



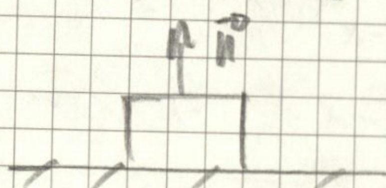
push



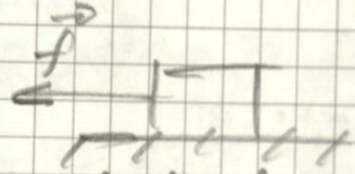
pull

- force is an interaction between two objects or between an object and its environment
- force is a vector quantity with magnitude and direction

Contact Forces:

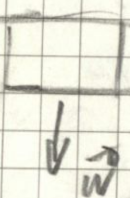


normal force
- when an object pushes on a surface (rests), the surface exerts a force ⊥ to the surface



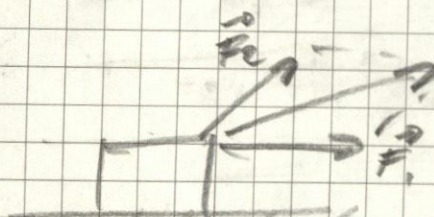
friction force
- a surface can also exert a force ∥ to the surface

tension force
- a pulling force exerted on an object by a rope, cord, etc.



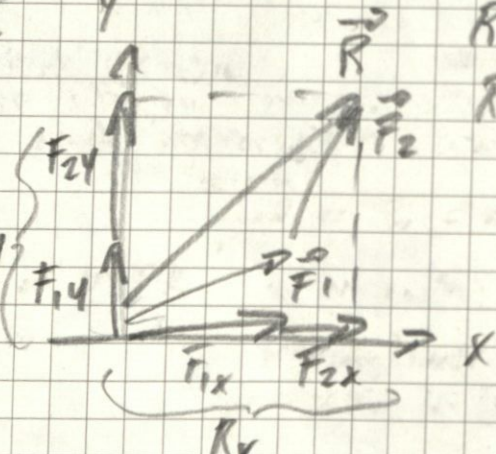
Weight: the pull of gravity is a long-range force (a force that acts over a distance)

SI: Newtons (N), magnitude of a force



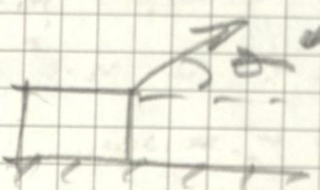
$$\vec{R} = \Sigma \vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \ldots$$

$$R_x = \Sigma F_x \qquad R_y = \Sigma F_y$$

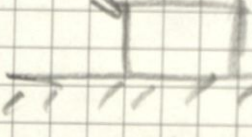


$$R = \sqrt{R_x^2 + R_y^2}$$

$$R = \sqrt{R_x^2 + R_y^2 + R_z^2}$$



θ above horizontal



θ below horizontal

4.2: Newton's First Law

"An object acted on by no net external force has a constant velocity (which may be zero) and zero acceleration."

inertia: tendency of an object to keep moving once it is set in motion

$\sum \vec{F} = \vec{F}_1 + \vec{F}_2$, but if $\sum \vec{F} = \vec{F}_1 + (-\vec{F}_1) = 0$, so object remains at rest

equilibrium: $\sum \vec{F} = 0$

* is only valid in the same inertial frame of reference

$\vec{v}_{P|A} = \vec{v}_{P|B} + \vec{v}_{B|A}$

4.3: Newton's Second Law

The magnitude of an object's acceleration $\vec{a}$ is directly proportional to the net external force $\sum \vec{F}$ acting on the object of mass m.

inertial mass m: quantitative measure of inertia

$$m = \frac{|\sum \vec{F}|}{a} \text{ or } |\sum \vec{F}| = ma \text{ or } a = \frac{|\sum \vec{F}|}{m}$$

SI: kilogram (kg) larger m means more resistance to acceleration

Also $1\,N = 1\,kg \cdot m/s^2$

$1\,N = \sum \vec{F}_{ext}$ where $\vec{a} = 1\,m/s^2$ for $m = 1\,kg$

$m_1 a_1 = m_2 a_2 \Rightarrow \frac{m_2}{m_1} = \frac{a_1}{a_2}$

"If a net external force acts on an object, that object accelerates. The direction of acceleration is the same as the direction of the net external force. The mass of the object times the acceleration equals the net external force vector."

$$\sum \vec{F} = m\vec{a} \Rightarrow \vec{a} = \frac{\sum \vec{F}}{m}$$

$\sum F_x = ma_x \quad \sum F_y = ma_y \quad \sum F_z = ma_z$

only refers to external forces
valid only when m is constant
valid in inertial frames of reference only

Note: $m\vec{a}$ is not a force, it is a result

Reading for Class #11: 2.8.24

4.4: Mass and Weight

different on different planets

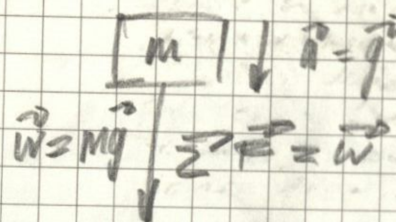
weight: gravitational force the earth exerts on an object
mass: inertial properties of an object
greater the mass, the greater the force needed to create acceleration, as per:

$$\sum \vec{F} = m\vec{a}$$

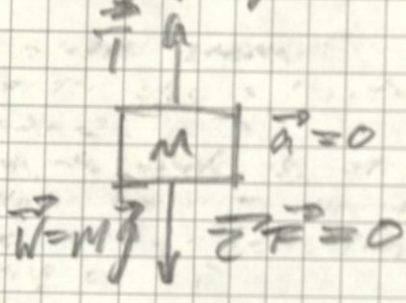
always the same

$\vec{w} = m\vec{g}$, or $w = mg$

falling object:



hanging object:



CAUTION: weight is always applied

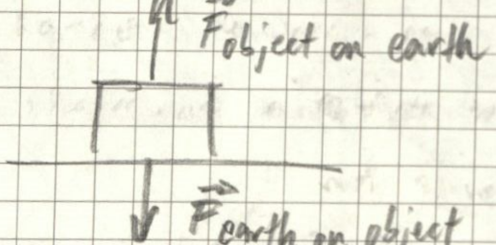
SI: $w = N$, $m = kg$

4.5: Newton's Third Law

"If an object A exerts a force on object B (an action), then object B exerts a force on object A (a reaction). These two forces have the same magnitude but are opposite direction. These forces act on different objects."

$$\vec{F}_{A\,on\,B} = -\vec{F}_{B\,on\,A}$$

CAUTION! A and B are different objects



$$\vec{F}_{O\,on\,E} = -\vec{F}_{E\,on\,O}$$

4.6. Free Body Diagrams

1. Newton's first and second laws apply to a specific object. You must decide at the beginning which object you are referring to to use:

   $\sum \vec{F} = \vec{0}$, for equilibrium situations

   $\sum \vec{F} = m\vec{a}$, for nonequilibrium situations

2. Only forces acting on the object matter. Once you select the object to analyze, you have to identify all the forces acting on it. Do not confuse the forces acting on an object with the forces an object exerts on another object. Only then can you use:

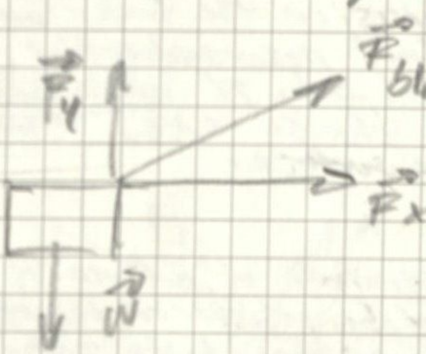
   $\sum \vec{F}$

3. Free Body Diagrams are essential to identify the relevant forces. A free-body-diagram shows the chosen object by itself, free of its surroundings, with vectors to show magnitudes and directions of all the forces that act on an object. Two forces in an action/reaction pair never appear in the same free-body-diagram. Same for forces that an object exerts on itself.

Note: When a problem involves more than one object, more than one free-body-diagram are required.

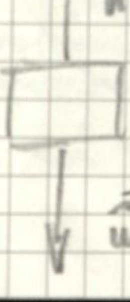
CAUTION: You must be able to answer "what other object is applying this force?" Avoid nonexistent forces such as "the force of acceleration".

Ex. Runner on starting block



"The force of the starting block on the runner has a vertical component that counteracts her weight and a horizontal component that accelerates her"

Ex. Somebody jumping



"To jump up, you push down on the floor, increasing the upward reaction force of $\vec{n}$ on you"

Reading for Class #12:

2.10.24

5.1. Using Newton's First Law: Particles in Equilibrium

IDENTIFY: the relevant concepts. Use Newton's first law for any problem in equilibrium (object at rest or moving w/ constant velocity).

If the problem involves more than one object and the objects interact w/ each other, also use Newton's third law. Relate the force at the first object to the second one and vice versa.

Identify the target variables. Common ones are magnitude and direction (angle) of a force, or components of a force.

SET UP: the problem using these steps

1. Draw a simple sketch of the physical situation, showing dimensions and angles
2. Draw a free-body-diagram for each object in equilibrium, expressing objects as large dots. Do not include other objects, such as a surface it may be resting on or rope that may be pulling on it.
3. Draw a force vector for each interaction with the object, labeling magnitude and angle. Include objects weight ($w = mg$). A surface in contact w/ object exerts a normal force perpendicular to the surface and a friction force parallel. A rope or chain exerts a pull (never push) in direction with its length.
4. Do not show any forces exerted by the object in the free-body-diagram. For each force that acts on the object, ask what other object causes that force (otherwise force may be nonexistent).
5. Set and include a coordinate axis in the free-body-diagram. Choose axis for each object independently. Label the positive direction for each object. If an object rests or slides on a tilted surface, choose axes that are parallel and perpendicular to the surface.

EXECUTE: the solution

1. Find components of each force. The magnitude of a force is always positive, but its components may be positive or negative.
2. Set sum of all x-components of forces equal to zero. In a separate equation, set all $F_y = 0$. Never add x and y components in a single equation.
3. Repeat 1. and 2. for each object in a problem. If objects interact with each other use Newton's Third Law.
4. Make sure you have as many independent equations as number of unknown quantities. Then solve to find target variables.

EVALUATE: your answers. Look at your results and ask why they make sense.

Reading for class #13: 2.13.24

5.2: Using Newton's Second Law: Dynamics of Particles

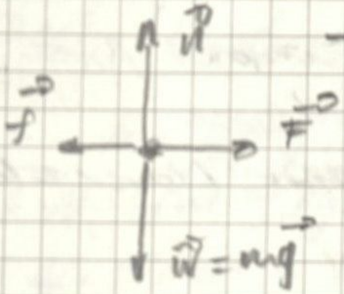
Dynamic Problems: apply $\sum \vec{F} = m\vec{a}$ to objects that are not in equilibrium and hence are accelerating

Same process as 5.1 except set $\sum \vec{F} = m\vec{a}$ and use kinematics equations

CAUTION! $m\vec{a}$ is not a force, never include it in the free-body-diagram

* $\vec{w}$ is a force, $\vec{w} = m\vec{g}$; and $\vec{n} = -\vec{w}$ *

ex.



$\sum \vec{F} = m\vec{a}$

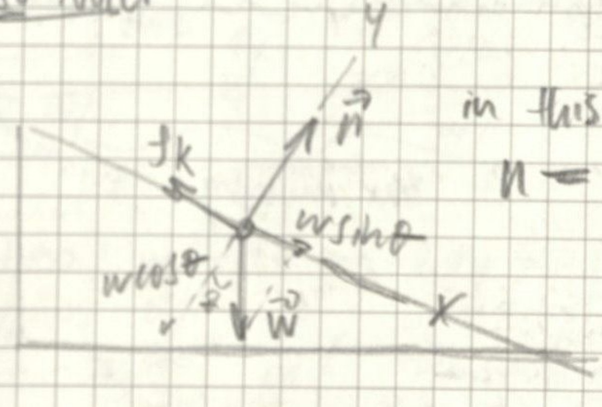
Apparent Weight and Weightlessness

$n = m(g + a_y)$ *bathroom scales show mass*

n — apparent weight
a_y — acceleration

ex. $a_y = -g$, free-fall. object seems to be weightless

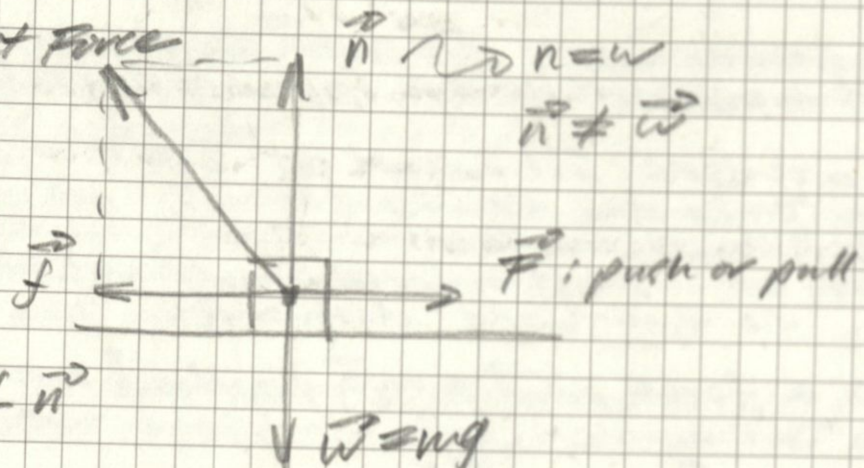
Also Notes



in this case, $n = w\cos\theta$

Reading for Class #14: 2.15.24

5.4: Friction Forces



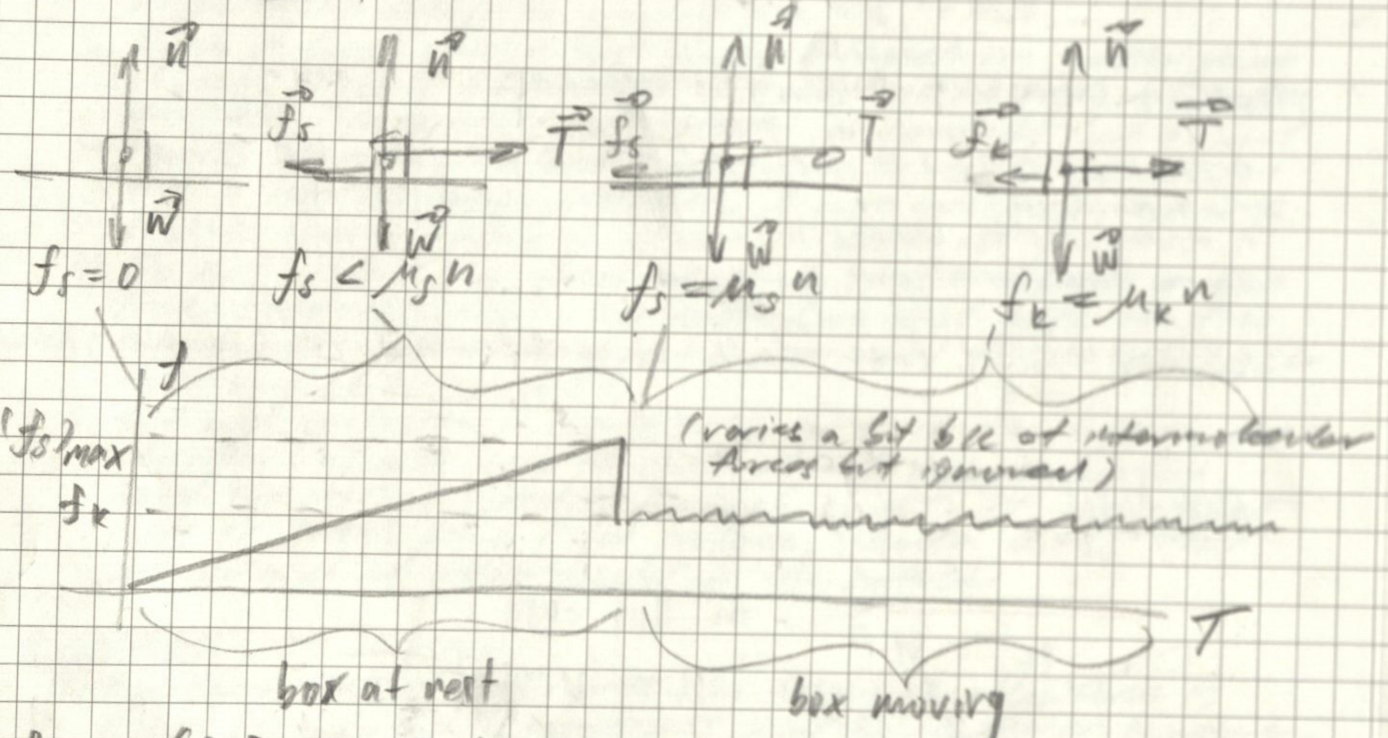
* $\vec{f}$ always $\perp$ $\vec{n}$

* $\vec{f}$ and $\vec{n}$ are components of a single contact force

$f_k = \mu_k n$ ← magnitude of normal force

← coefficient of kinetic friction (lower = less friction)

magnitude of kinetic friction



$f_s \le (f_s)_{max} = \mu_s n$

← max static friction

magnitude of static friction force

μ_r: coefficient of rolling friction or "tractive resistance"

horizontal force needed for constant speed on a flat surface divided by the upward normal force exerted by the surface

Fluid Resistance and Terminal Speed:

fluid: gas or liquid, by Newton's third law it pushes back on objects that push it

direction → opposite of direction of objects velocity relative to the fluid

magnitude → increases with speed of the object

* $\vec{f}$ is usually independent of speed *

$f = kv$ (fluid resistance at low speed)

↑ proportionality constant

$f = Dv^2$ (fluid resistance at high speed)

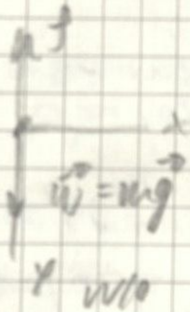
↑ air drag, depends on size and shape of object

SI: k: $N \cdot s/m = kg/s$

D: $N \cdot s^2/m^2 = kg/m$

* objects experiencing fluid resistance cannot have constant acceleration, so kinematic equations cannot be used *

Ex. ball dropping in oil

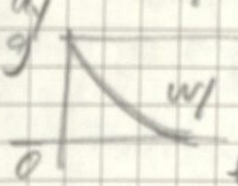


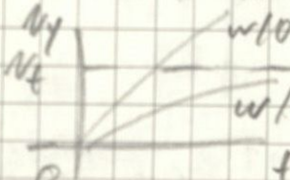
$\sum F_y = mg + (-kv_y) = ma_y$

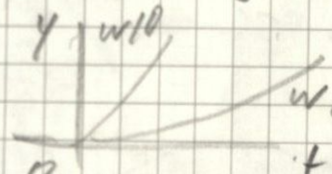
eventually $mg - kv_y = 0$ and $a_y = 0$

is terminal speed: $v_t = \frac{mg}{k}$

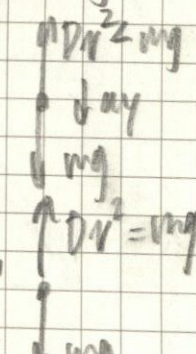
(for fluid resistance $f = kv$)







Ex. falling w/ drag



Derivation: $m\frac{dv_y}{dt} = mg - kv_y \Rightarrow \int_0^{v} \frac{dv_y}{v_y - v_t} = -\frac{k}{m}\int_0^t dt$

$\Rightarrow \ln \frac{v_t - v_y}{v_t} = -\frac{k}{m}t \Rightarrow 1 - \frac{v_y}{v_t} = e^{-(k/m)t}$

$\Rightarrow v_y = v_t\left[1 - e^{-(k/m)t}\right]$, $a_y = g e^{-(k/m)t}$

and $y = v_t\left[t - \frac{m}{k}\left(1 - e^{-(k/m)t}\right)\right]$

Also, $v_t = \sqrt{\frac{mg}{D}}$ (terminal speed, fluid resistance $f = Dv^2$)

* parabolic projectile motion has limited range and max height, and is no longer parabolid w/ air resistance *

More Fluid and Air Resistance:

2.19.24

What if $v(0) \neq 0$?:

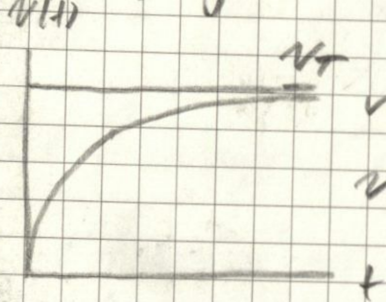
Generalized Solution

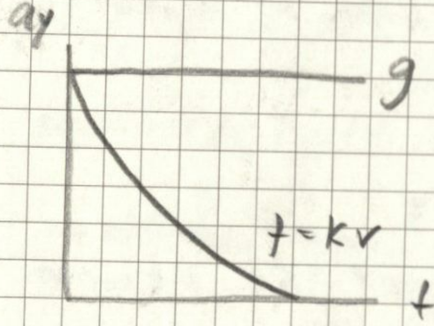
$$v(t) = v_T\left(A + B e^{(-k/m)t}\right)$$

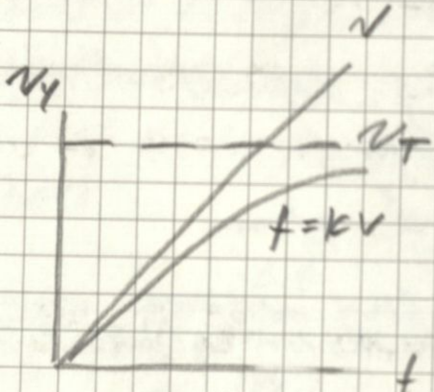
↳ Find A & B by checking $t=0$ and $t=\infty$

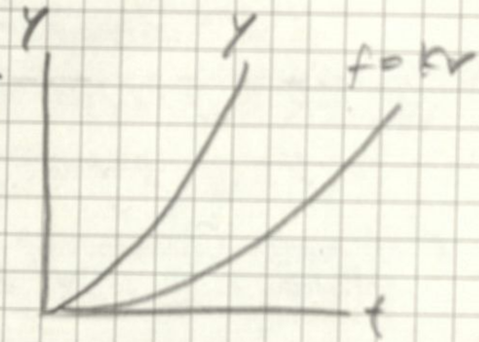
$$v(0) = v_T(A + Be^{0}) = v_T(A+B)$$

$$v(\infty) = v_T(A + Be^{-\infty}) = v_T(A)$$









$$y = v_T\left[t - \frac{m}{k}\left(1 - e^{-(k/m)t}\right)\right]$$

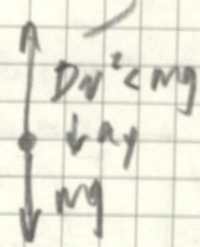
$$v_y = v_T\left[1 - e^{-(k/m)t}\right]$$

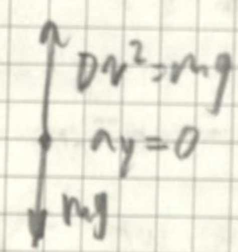
$$a_y = g e^{-(k/m)t}$$

assumes $v(0) = 0$

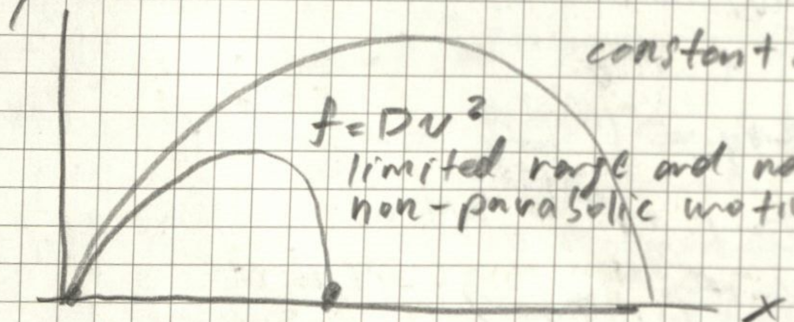
For $f = kv$, $v_T = \frac{mg}{k}$

For $f = Dv^2$, $v_T = \sqrt{\frac{mg}{D}}$

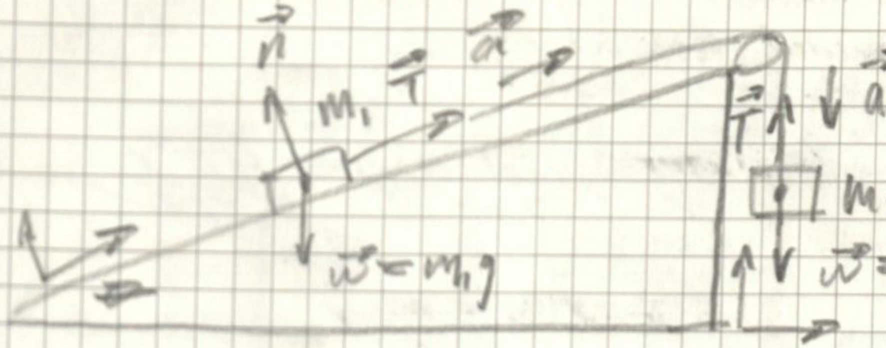




constant a, parabolic motion

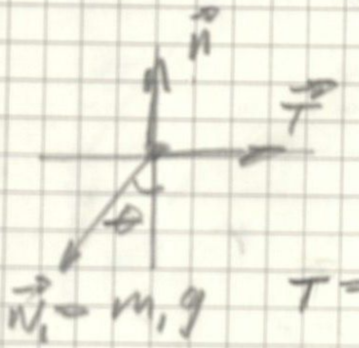


Equilibrium Tension on an inclined plane



or $\Sigma F_y = -ma$

Solve a in terms of m_1, m_2, and θ.
What values of m_1 and m_2 give $a = 0$?

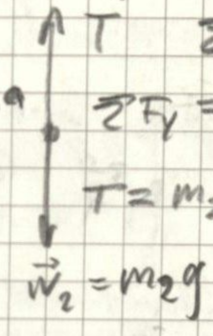


$\Sigma \vec{F} = m_1 \vec{a}$

$\Sigma F_x = T - m_1 g \sin\theta = m_1 a$

$\Sigma F_y = n - m_1 g \cos\theta = 0$

$T = m_1 g \sin\theta + m_1 a$



$\Sigma \vec{F} = m_2 \vec{a}$

$\Sigma F_y = T - m_2 g = -m_2 a$

$T = m_2 g - m_2 a$

$\Rightarrow m_1 a + m_1 g \sin\theta = m_2 g - m_2 a$

$$\Rightarrow a = \frac{g(m_2 - m_1 \sin\theta)}{m_1 + m_2}$$

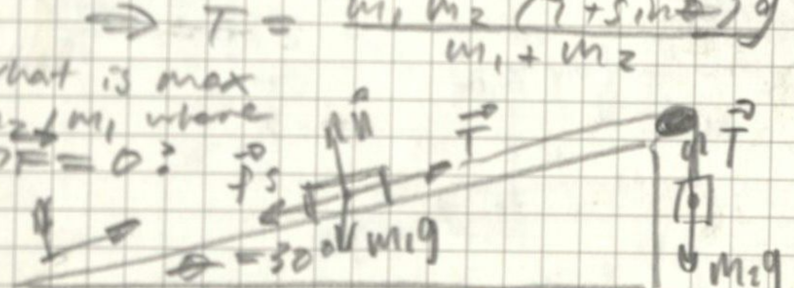
for $a = 0$; $m_2 - m_1 \sin\theta = 0 \Rightarrow \frac{m_2}{m_1} = \sin\theta$

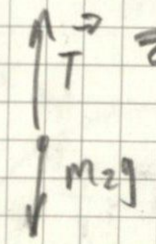
Find T?

$$T = m_2 g - m_2 a = m_2 g - m_2 g \left(\frac{m_2 - m_1 \sin\theta}{m_1 + m_2} \right)$$

$$\Rightarrow T = \frac{m_1 m_2 (1 + \sin\theta) g}{m_1 + m_2}$$

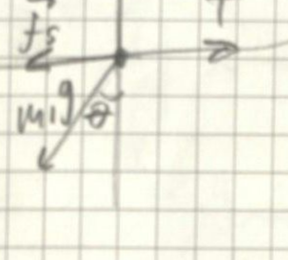
What is max m_2 / m_1 where $\Sigma F = 0$?





$\Sigma F_y = T - m_2 g = 0$

$T = m_2 g$



$\Sigma F_y = n - m_1 g \cos\theta = 0$

$\Sigma F_x = T - \mu_s n - m_1 g \sin\theta = 0$

$\Sigma F_x = m_2 g - \mu_s m_1 g \cos\theta - m_1 g \sin\theta = 0$

$$\Rightarrow \frac{m_2}{m_1} = \mu_s \cos\theta + \sin\theta$$

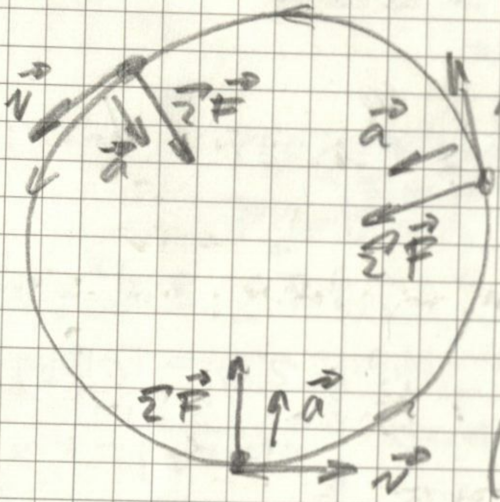
Reading for Class #16:

2.20.24

5.4: Dynamics of Circular Motion

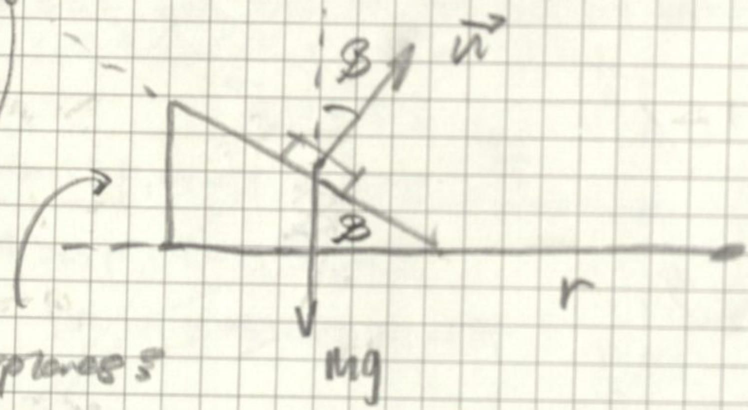
Remember:

$$a_{rad} = a_c = \frac{v^2}{r}, \quad T = \frac{2\pi r}{v}, \quad \text{and} \quad a_{rad} = \frac{4\pi^2 r}{T^2}$$

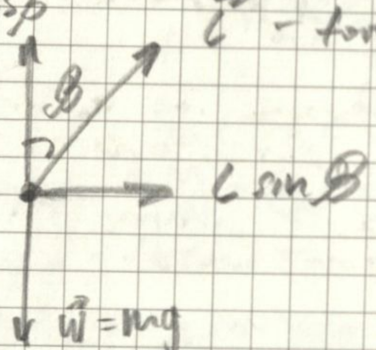


$$\Sigma F = ma_c = m\frac{v^2}{r}$$

* anything involving v is never a force *

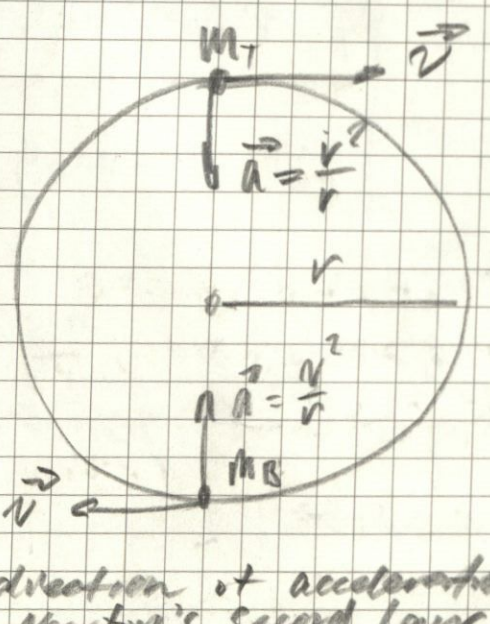


Banked Curves and Airplanes:

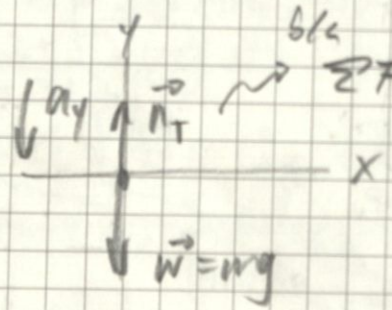


- $L\cos\beta = mg$
- $L\sin\beta = \frac{mv^2}{r}$
- $\vec{n}$ is apparent weight in problem

Motion in a Vertical Circle:

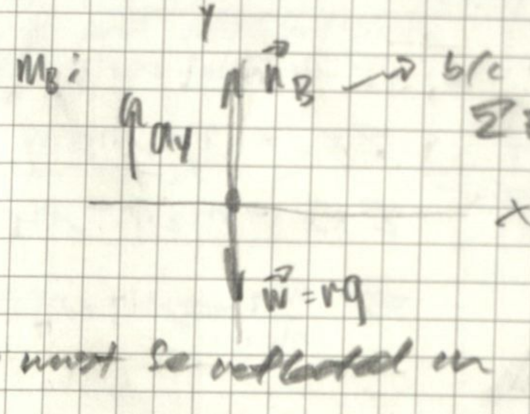


m_T:



→ b/c $\Sigma F_y = -\frac{mv^2}{r}$

m_B:

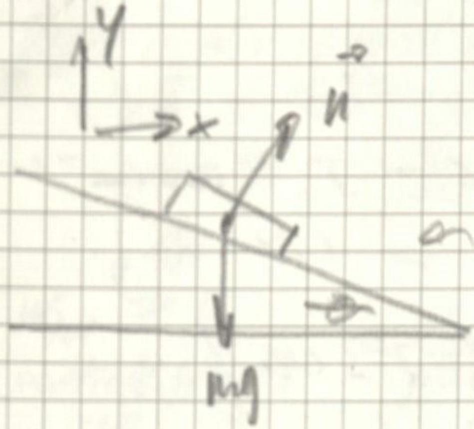


→ b/c $\Sigma F_y = \frac{mv^2}{r}$

* direction of acceleration must be reflected in Newton's Second Law

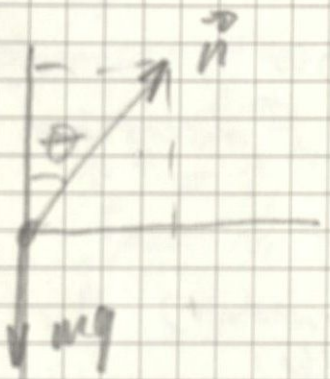
The Banked Curve

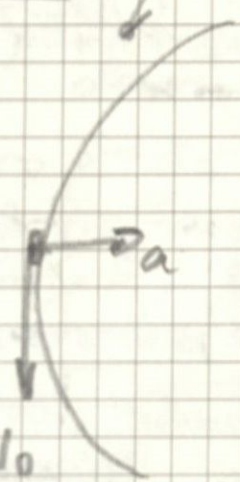
2.21.24



What is the fastest speed where the block will not move up or down?

* traveling in a circle





$$\sum F_y = n\cos\theta - mg = 0$$

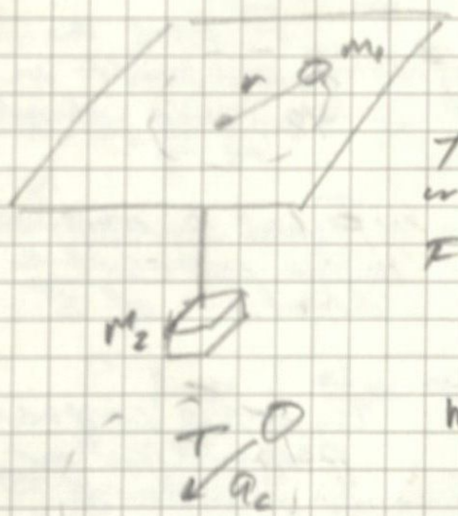
$$\Rightarrow n = \frac{mg}{\cos\theta}$$

$$\sum F_x = n\sin\theta = ma_x = \frac{mv^2}{r}$$

$$= mg\frac{\sin\theta}{\cos\theta} = m\frac{v^2}{r}$$

$$g\tan\theta = \frac{v^2}{r}$$

$$v = \sqrt{gr\tan\theta}$$



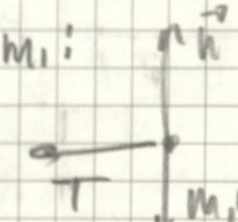
Puck m_1 moves in r_0 with v_0 on a level, frictionless table.

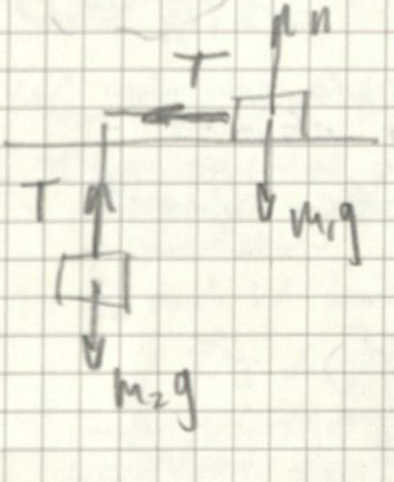
The puck now moves $v = 2v_0$, with the same m_2.

Find the new a & r

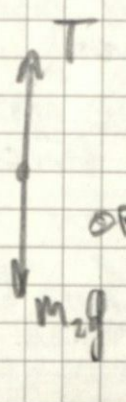
$$\sum F_x = T = m_1\frac{v_0^2}{r_0}$$

m_1:





m_2:



$$\sum F_y = T - m_2 g = 0$$

$$\Rightarrow T = m_2 g = m_1\frac{v_0^2}{r_0}$$

$$\Rightarrow r_0 = \frac{m_1 v_0^2}{m_2 g}$$

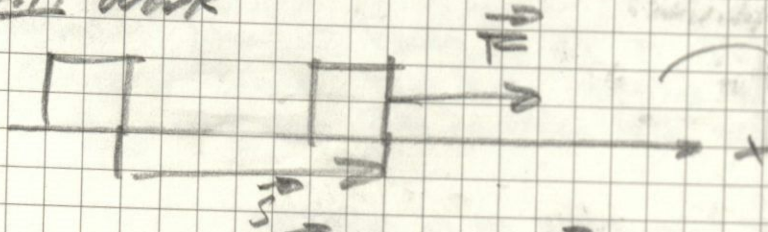
OR

$$\Rightarrow m_2 g = m_1\frac{(2v_0)^2}{r_0}$$

$$\Rightarrow r = 4\left[\frac{m_1 v_0^2}{m_2 g}\right]$$

Reading for Class #17: 2.22.24

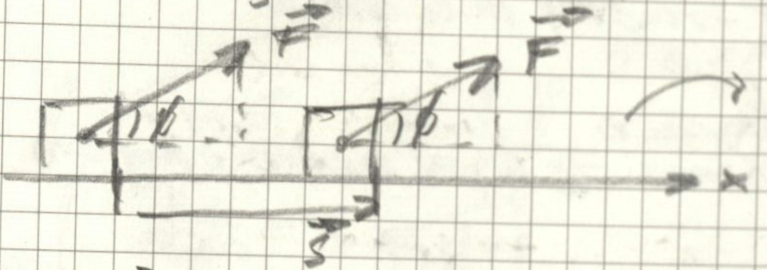
6.1: Work



$\curvearrowright W = Fs$

SI: J, "joule"

$= 1\,N \cdot m$



$\curvearrowright W = Fs\cos\phi$

* all $\vec{F}$ are constant for now, non-constant $\vec{F}$ discussed later

For any vector: Note: Work is a scalar

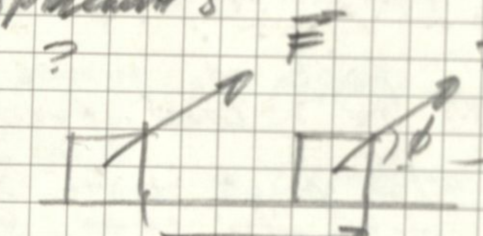
$W = \vec{F} \cdot \vec{s}$ → scalar product (dot product)

↳ work done on a particle by constant force $\vec{F}$ along straight-line displacement $\vec{s}$

essentially: $W = \int \vec{F} \cdot d\vec{r}$

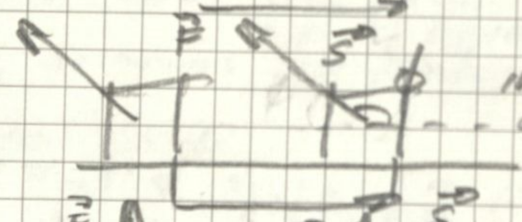
Work has direction? ?

$W = F\cos\phi\, s$
work is positive



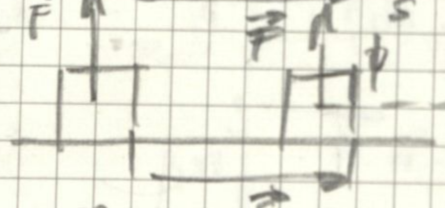
$\vec{F}$ has a component in the same direction as $\vec{s}$

$W = F\cos\phi\, s$
work is negative



$\vec{F}$ has a component in the opposite direction of $\vec{s}$

$W = F\cos\phi\, s$
work is 0



$\vec{F}$ has a component perpendicular to direction of $\vec{s}$

$\cos 90° = 0$

* you can expend "energy" or apply a force to an object and do no work on that object

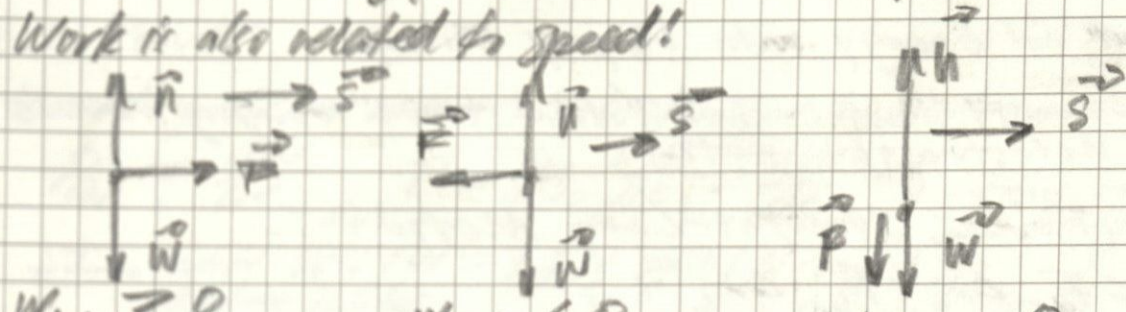
CAUTION: work is always done on a specific object by a specific force

Total Work:

$W_{tot} = \sum W_i$, or $W_{tot} = (\sum \vec{F}) \cdot \vec{s}$

6-2: Kinetic Energy and the Work-Energy Theorem

Work is also related to speed!



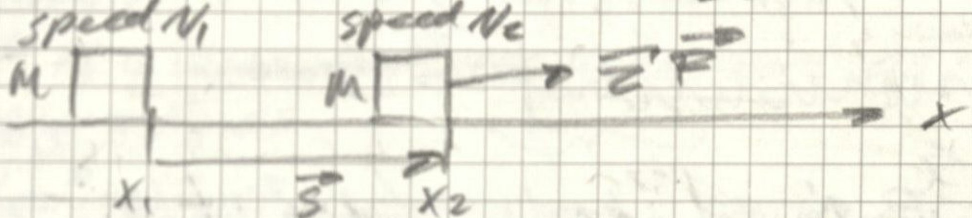
$W_{tot} > 0$ particle speeds up

$W_{tot} < 0$ particle slows down

$W_{tot} = 0$ particle's speed is unchanged

<u>Suppose</u>: $F = ma_x$, $s = x_2 - x_1$, and speed changes from v_1 to v_2

$$v_2^2 = v_1^2 + 2a_x s, \quad a_x = \frac{v_2^2 - v_1^2}{2s}$$

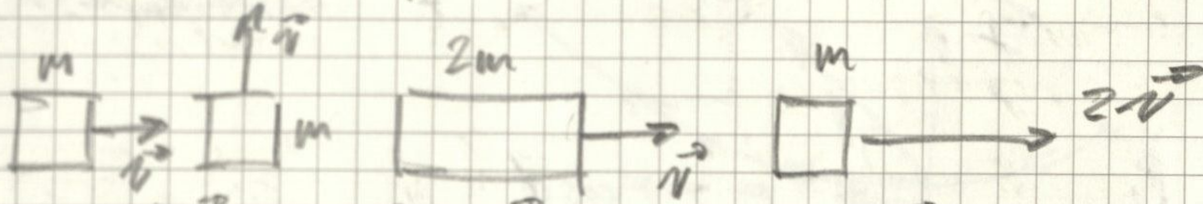


$$F = ma_x = m\frac{v_2^2 - v_1^2}{2s}, \quad Fs = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$$

* from kinematic equations

$K = \frac{1}{2}mv^2 \rightarrow W_{tot} = K - 0 = K$ "total work needed to accelerate from rest to present speed"

kinetic energy of a particle



same m & $\vec{v}$, different direction, same K

$2m$, $\vec{v}$, K doubled

m, $2\vec{v}$, K doubled

$W_{tot} = \Delta K = K_2 - K_1$: Work-Energy Theorem

* can only tell us info on change in speed Δv, <u>not</u> change in velocity $\Delta\vec{v}$; since K doesn't depend on direction

K and W have same <u>SI</u>:

$$1\,J = 1\,N\cdot m = 1(kg\cdot m/s^2)m = 1\,kg\cdot m^2/s^2$$

* since Newton's Laws are used, only applicable in an inertial frame of reference

Reading for Class #18: 2.25.24

6.3: Work and Energy with Varying Forces

* work-energy theorem holds true with varying forces and not-straight paths

Varying Force, Straight-Line Motion:

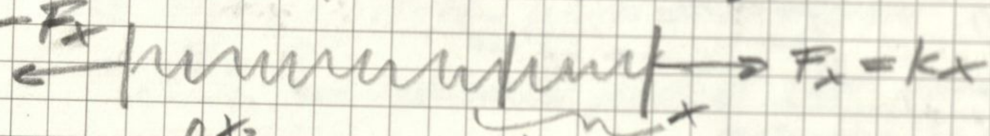
$W \approx F_{ax}\Delta x_a + F_{bx}\Delta x_b + \ldots$

$W = \int_{x_1}^{x_2} F_x\, dx$

if F_x is constant:

$W = F_x \int_{x_1}^{x_2} dx = F_x (x_2 - x_1) = F_x \Delta x$

$F_x = kx$ (force required to stretch a spring) (non-constant)



$W = \int_{x_1}^{x_2} F_x\, dx = \int_{x_1}^{x_2} kx\, dx = \frac{1}{2}kx_2^2 - \frac{1}{2}kx_1^2 \overset{x_1=0}{\Longrightarrow} \frac{1}{2}kx^2$

↳ work done on a spring, work done by a spring is $-W$

* $W_{tot} = \Delta K$ still holds

Motion Along a Curve:

$W = \int_{P_1}^{P_2} \vec{F}\cdot d\vec{l} = \int_{P_1}^{P_2} F\cos\phi\, dl = \int_{P_1}^{P_2} F_\parallel\, dl$

↳ work done on a particle by a varying force $\vec{F}$ along a curved path l

* $W_{tot} = \Delta K$ still holds

* $F_\perp$ has no effect on particle speed (or work), only on particle direction

Reading for Class #19: 2.27.24

6.4: Power

* work makes no reference to the passage of time

$P_{av} = \frac{\Delta W}{\Delta t}$, $P = \lim_{\Delta t \to 0} \frac{\Delta W}{\Delta t} = \frac{dW}{dt}$ ← time rate of doing work

SI: watt W: $1 W = 1 J/s = 1 (N \cdot m)/s$
$1 kW = 10^3 W$
$1 MW = 10^6 W$
$\hookrightarrow 1 (kg \cdot \frac{m}{s^2} \cdot m)/s$

Horsepower: $1 hp = 746 W = 0.746 kW$

Electrical Energy: $kW \cdot h$, "total work done in 1 hour when the power is 1 kW"
$\Rightarrow (10^3 J/s)(3600 s) = 3.6 MJ$ → unit of work or energy, not power

In terms of $\vec{F}$ and v:

if $F_{\parallel}$ is tangent to $\vec{F}$ over displacement $\Delta \vec{s}$,
$\Delta W = F_{\parallel} \Delta s$

$P_{av} = \frac{F_{\parallel} \Delta s}{\Delta t} = F_{\parallel} \frac{\Delta s}{\Delta t} = F_{\parallel} v_{av}$

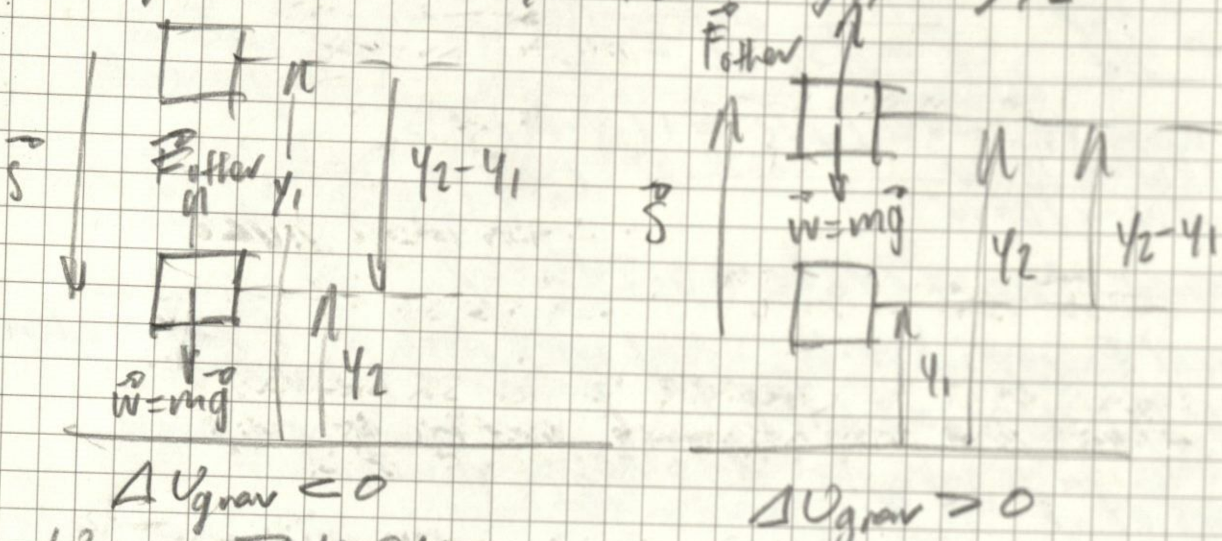
as $\Delta t \to 0$:
$P = F_{\parallel} v \Rightarrow P = \vec{F} \cdot \vec{v}$ ← velocity of particle

instantaneous power for a force doing work on a particle; force acting on particle

Reading for Class #20: 2.29.24

7.1: Gravitational Potential Energy

$W_{grav} = Fs = w(y_1 - y_2) = mgy_1 - mgy_2$



$U_{grav} = mgy$

$W_{grav} = mgy_1 - mgy_2 = U_{grav,1} - U_{grav,2} = -\Delta U_{grav}$

Suppose $\vec{F}_{other} = 0$

$W_{tot} = \Delta K = K_2 - K_1$, $W_{tot} = W_{grav} = -\Delta U_{grav}$

$\Delta K = -\Delta U_{grav} \Rightarrow K_2 - K_1 = U_{grav,1} - U_{grav,2}$

$\Rightarrow K_1 + U_{grav,1} = K_2 + U_{grav,2}$

$\Rightarrow \frac{1}{2}mv_0^2 + mgy_0 = \frac{1}{2}mv_f^2 + mgy_f$

$E = K + U_{grav} = \text{constant}$ (if $W_{tot} = W_{grav}$)

$\hookrightarrow$ total mechanical energy of system

CAUTION: you can choose "zero" height to be wherever is convenient

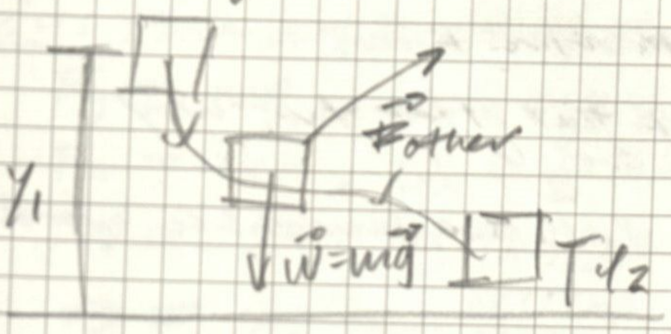
Suppose $\vec{F}_{other} \neq 0$:

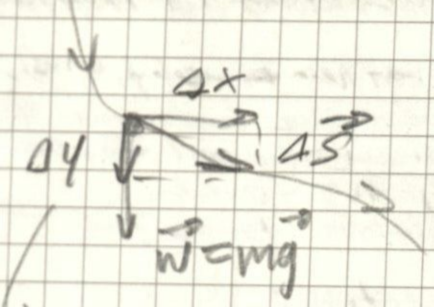
$W_{other} + W_{grav} = K_2 - K_1$, $W_{grav} = -\Delta U_{grav}$

$W_{other} + U_{grav,1} - U_{grav,2} = K_2 - K_1$

$\Rightarrow K_1 + U_{grav,1} + W_{other} = K_2 + U_{grav,2}$

U_{grav} Along a Curved Path:





in this case: $\Delta y < 0$

$$W_{grav} = \vec{w}\cdot\Delta\vec{s} = -mg\hat{j}\cdot(\Delta x\hat{i}+\Delta y\hat{j}) = -mg\Delta y$$

* the work done by gravity is the same as though the object had been displaced vertically Δy with <u>no</u> horizontal displacement Δx

$$W_{grav} = -mg(y_2-y_1) = mgy_1 - mgy_2 = U_{grav,1} - U_{grav,2}$$

<u>7.2</u>: Elastic Potential Energy

For ideal springs: $F = kx$

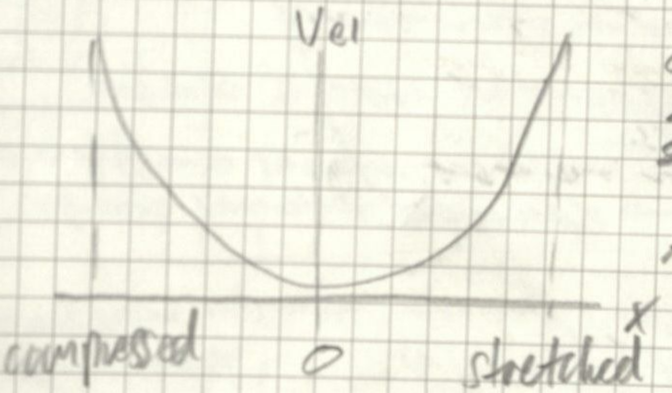
$W = \frac{1}{2}kx_2^2 - \frac{1}{2}kx_1^2$ (work done <u>on</u> a spring)

$W_{el} = \frac{1}{2}kx_1^2 - \frac{1}{2}kx_2^2$ (work done <u>by</u> a spring)

↳ elastic

$\Rightarrow U_{el} = \frac{1}{2}kx^2$ ($x>0$ if stretched, $x<0$ if compressed)

$W_{el} = U_{el,1} - U_{el,2} = -\Delta U_{el}$



<u>CAUTION</u>: in this case, $x=0$ <u>must</u> be where the spring is neither stretched or compressed.

<u>Also</u>: if U_{el} is the only force

$W_{tot} = W_{el} = U_{el,1} - U_{el,2}$

$\Rightarrow K_1 + U_{el,1} = K_2 + U_{el,2}$

Situations with both U_{grav} and U_{el}:

if $W_{tot} = W_{grav} + W_{el} + W_{other}$

$\Rightarrow W_{grav} + W_{el} + W_{other} = K_2 - K_1$

$\Rightarrow K_1 + U_{grav,1} + U_{el,1} + W_{other} = K_2 + U_{grav,2} + U_{el,2}$

$\Rightarrow K_1 + U_1 + W_{other} = K_2 + U_2$

Reading for Class #21

3. 3. 24

7.3: Conservative and Nonconservative Forces

Conservative Forces: a force that offers two-way conversion between K and U
- gravitational force
- spring force
- electric force?

* all have reversable work

Properties:
1. It can be expressed as the difference btwn initial and final values of a potential-energy function
2. It is reversible
3. It is independent of path (only depends on starting and ending points)
4. when the starting and ending points are the same, $W_{tot} = 0$

When all forces are conservative,

$E = K + U$ is constant
→ total mechanical energy

Nonconservative Forces: cannot be represented as a potential-energy function
- friction force
- fluid-resistance force

Dissipative Force: loses mechanical energy
* forces that increase mechanical energy also exist

Law of Conservation of Energy:

Nonconservative Forces cannot be expressed with potential energy, but can be described in other terms

internal energy: energy associated with a change of state of its materials
- raising temperature → increases internal energy
- lowering temperature → decreases internal energy

$\Delta U_{int} = -W_{other}$

$\Rightarrow K_1 + U_1 - \Delta U_{int} = K_2 + U_2$

$\Rightarrow \Delta K + \Delta U + \Delta U_{int} = 0$

* energy is never created or destroyed; only changed in form *

Reading for Class #22: 3.5.24

2.4: Force and Potential Energy

$F_g = -mg, \quad U_g = mgy$

$F_x = -kx, \quad U_x = \frac{1}{2}kx^2$

$W = -\Delta U, \quad F_x(x)\Delta x = -\Delta U$

$\Rightarrow F_x(x) = -\frac{dU(x)}{dx}$

↳ 1D motion

* a conservative force always acts to push the system toward lower potential energy

$\Rightarrow F_x(x) = -\frac{d}{dx}\left(\frac{1}{2}kx^2\right) = -kx$

$\Rightarrow F_y(x) = -\frac{d}{dy}(mgy) = -mg$

in 3D:

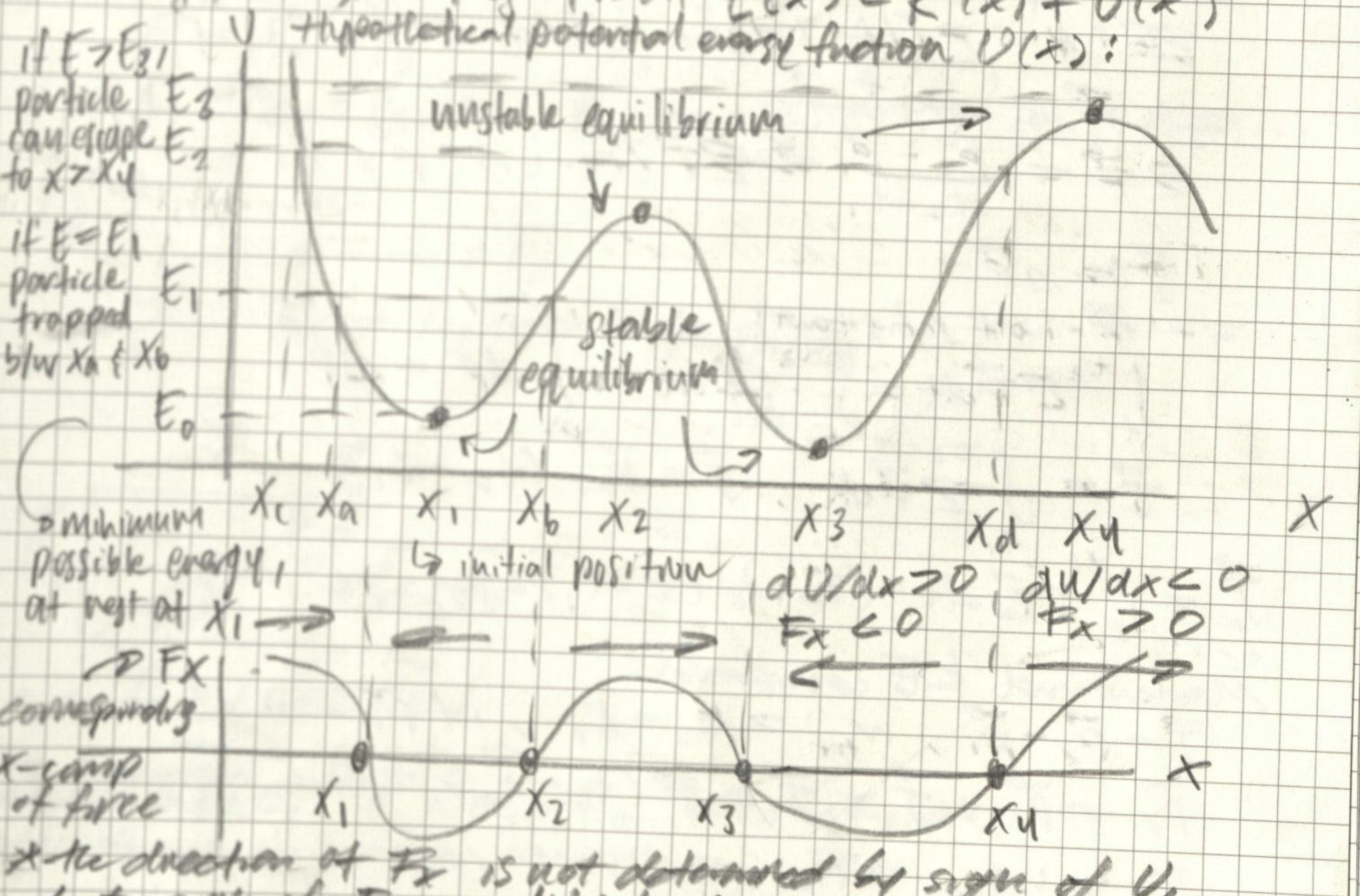
$F_x = -\frac{\partial U}{\partial x} \quad F_y = -\frac{\partial U}{\partial y} \quad F_z = -\frac{\partial U}{\partial z}$

$\vec{F} = -\left(\frac{\partial U}{\partial x}\hat{i} + \frac{\partial U}{\partial y}\hat{j} + \frac{\partial U}{\partial z}\hat{k}\right) = -\vec{\nabla} U$

2.5: Energy Diagrams

energy diagram: graph of $E(x) = K(x) + U(x)$

hypothetical potential energy function $U(x)$:



* the direction of F_x is not determined by sign of U, but sign of $F_x = -dU/dx$ (difference of two points of U)

Reading for Class #23: 3.7.24

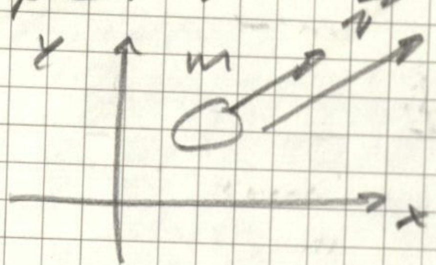
8.1: Moment and Impulse

$\sum \vec{F} = m\vec{a} = m\frac{d\vec{v}}{dt} = \frac{d}{dt} m\vec{v}$

$\vec{p} = m\vec{v}$

→ moment of particle (vector)

$p = mv$



* moment and velocity in same direction

$p_x = mv_x,\ p_y = mv_y,\ p_z = mv_z$

SI: kg·m/s, plural "moments"

$\sum \vec{F} = \frac{d\vec{p}}{dt}$ → rate of change of particle's moment

→ newton's third law in terms of momentum

* only valid in inertial frames of reference

$\vec{J} = \sum \vec{F}(t_2 - t_1) = \sum \vec{F} \Delta t$

→ Impulse of constant net external force

SI: $N \cdot s = (1 kg \cdot m/s^2) \cdot s = kg \cdot m/s$

If $\vec{F}$ is constant:

$\sum \vec{F} = \frac{\vec{p}_2 - \vec{p}_1}{t_2 - t_1} \Rightarrow \sum \vec{F}(t_2 - t_1) = \vec{p}_2 - \vec{p}_1$

$\vec{J} = \Delta \vec{p}$

If $\vec{F}$ not constant

$\int_{t_1}^{t_2} \sum \vec{F} dt = \int_{t_1}^{t_2} \frac{d\vec{p}}{dt} dt = \int_{p_1}^{p_2} d\vec{p} = \Delta \vec{p}$

$\vec{J} = \int_{t_1}^{t_2} \sum \vec{F} dt,\ \vec{J} = \vec{F}_{av}(t_2 - t_1)$

$\vec{J}_x = \int_{t_1}^{t_2} \sum F_x dt = (F_{av})_x (t_2 - t_1) = p_{2x} - p_{1x} = mv_{2x} - mv_{1x}$

...

Moment and KE compared:

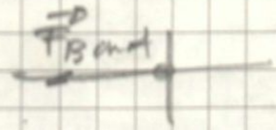
$\vec{J} = \vec{p}_2 - \vec{p}_1,\ W_{tot} = K_2 - K_1$, if $\vec{p}_1$ and $K_1 = 0$:

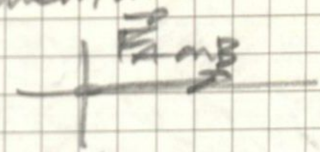
$\vec{p}_2 = \vec{p}_1 + \vec{J} = \vec{J} = \vec{F}(t_2 - t_1)$ → time required to accelerate

$K_2 = W_{tot} = Fs$ → distance required to accelerate

* Both impulse—momentum and work-energy theorems founded with Newton's laws are integral principals, whereas $\Sigma \vec{F} = m\vec{a}$ or $\Sigma \vec{F} = d\vec{p}/dt$ itself is a differential principle *

8.2 : Conservation of Momentum



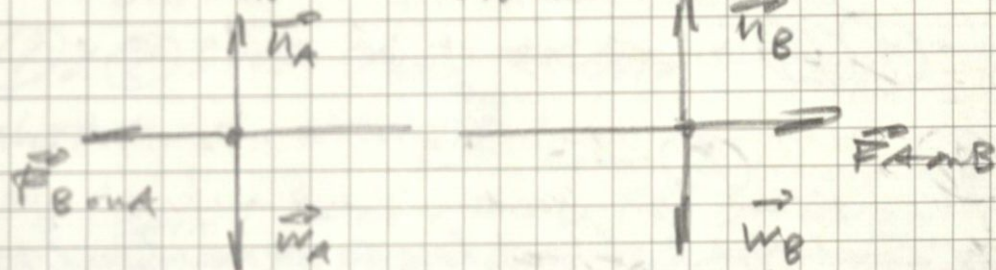


* perfect collisions are action—reaction pairs

$$\vec{F}_{B\,on\,A} = \frac{d\vec{p}_A}{dt}, \quad \vec{F}_{A\,on\,B} = \frac{d\vec{p}_B}{dt}$$

$$\vec{F}_{B\,on\,A} = -\vec{F}_{A\,on\,B} \Rightarrow \vec{F}_{B\,on\,A} + \vec{F}_{A\,on\,B} = 0$$

$$\Rightarrow \frac{d\vec{p}_A}{dt} + \frac{d\vec{p}_B}{dt} = \frac{d(\vec{p}_A + \vec{p}_B)}{dt} = \frac{d\vec{P}}{dt} = 0$$

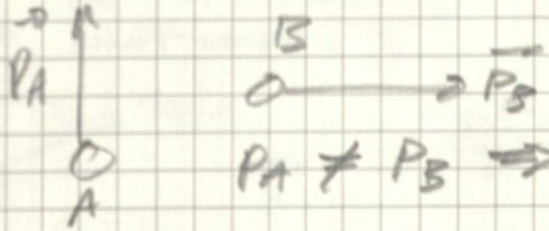


Conservation of Momentum! if:

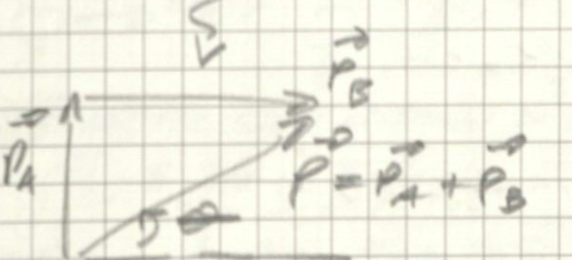
$$\Sigma \vec{F}_A + \Sigma \vec{F}_B = 0, \ \vec{P} \text{ is conserved}$$

$$\vec{P} = \vec{p}_A + \vec{p}_B + \dots = m_A \vec{v}_A + m_B \vec{v}_B + \dots$$

$$P_x = p_{Ax} + p_{Bx} + \dots, \ P_y = p_{Ay} + p_{By} + \dots, \ P_z = p_{Az} + p_{Bz} + \dots$$



$$p_A \neq p_B \Rightarrow P \neq p_A + p_B$$



$$P = |\vec{p}_A + \vec{p}_B|$$

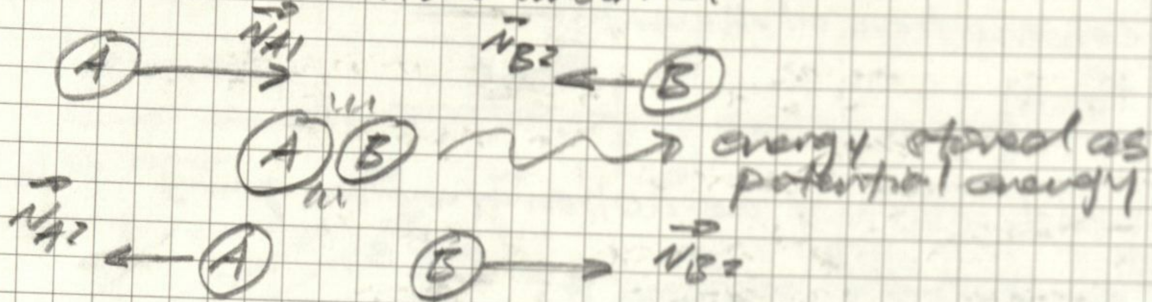
$= \_\_$ m/s at θ

if $\vec{F} = 0$, $P_x = P_y = P_z = $ constant

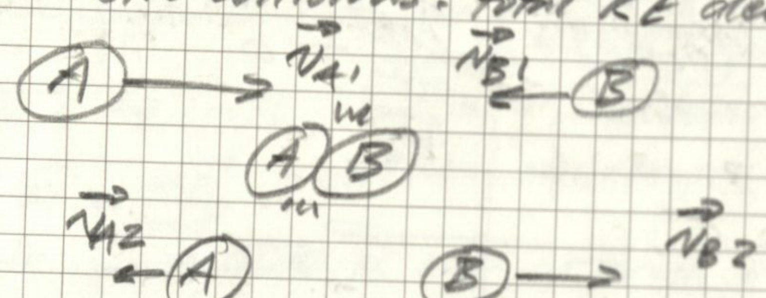
Reading for Class #24: 3.17.24

8.3. Moment Conservation and Collisions

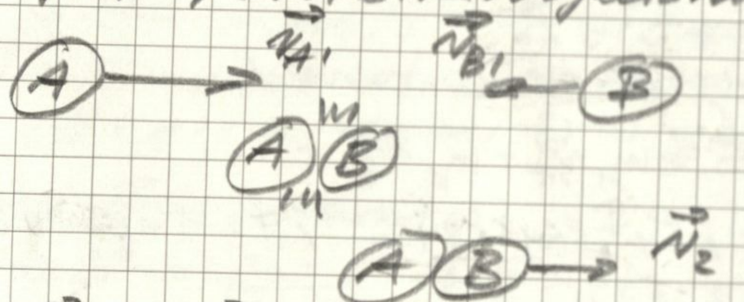
Elastic Collisions: KE is conserved



Inelastic Collisions: total KE decreases



Completely Inelastic: objects have same final velocity



conservation of momentum

$\vec{v}_{A2} = \vec{v}_{B2} = \vec{v}_2$

$m_A \vec{v}_{A1} + m_B \vec{v}_{B1} = (m_A + m_B)\vec{v}_2$

Verify with KE!

$K_1 = \frac{1}{2} m_A v_{A1x}^2$

$K_2 = \frac{1}{2}(m_A + m_B) v_{2x}^2$

8.4. Elastic Collisions

In 1-D, along the x-axis, we have:

$$\frac{1}{2} m_A v_{A1x}^2 + \frac{1}{2} m_B v_{B1x}^2 = \frac{1}{2} m_A v_{A2x}^2 + \frac{1}{2} m_B v_{B2x}^2$$

and:

$$m_A v_{A1x} + m_B v_{B1x} = m_A v_{A2x} + m_B v_{B2x}$$

$\underline{v_{B1x} = 0}$: object B is a target for object A to hit:

$$\frac{1}{2} m_A v_{A1x}^2 = \frac{1}{2} m_A v_{A2x}^2 + \frac{1}{2} m_B v_{B2x}^2 \,,$$

$$m_A v_{A1x} = m_A v_{A2x} + m_B v_{B2x}$$

1 $\Rightarrow m_B v_{B2x}^2 = m_A (v_{A1x}^2 - v_{A2x}^2)$

$= m_A (v_{A1x} - v_{A2x})(v_{A1x} + v_{A2x})$

2 $\Rightarrow m_B v_{B2x} = m_A (v_{A1x} - v_{A2x})$

3: 1 / 2: $\Rightarrow v_{B2x} = v_{A1x} + v_{A2x}$

Substitute into 2:

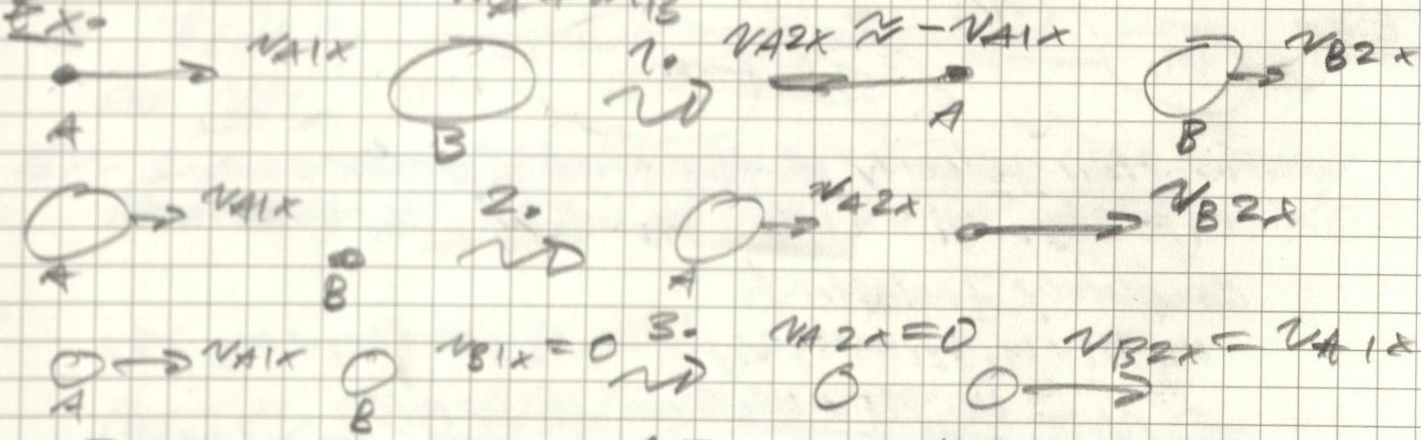
$$m_B (v_{A1x} + v_{A2x}) = m_A (v_{A1x} - v_{A2x})$$

$$\Rightarrow v_{A2x} = \frac{m_A - m_B}{m_A + m_B} v_{A1x}$$

Substitute into 3:

$$\Rightarrow v_{B2x} = \frac{2 m_A}{m_A + m_B} v_{A1x}$$

Ex:



Elastic Collisions and Relative Velocity:

3: $v_{A1x} = v_{B2x} - v_{A2x}$

$-v_{B/A}$ before collision $= v_{B/A}$ after collision

if $\underline{v_{B1x} \neq 0}$:

$$v_{B2x} - v_{A2x} = -(v_{B1x} - v_{A1x})$$

* relative velocities have same magnitude before and after collision

Lecture Notes: Collisions 3.14.24

$\sum \vec{P} = \vec{P}_A + \vec{P}_B + \ldots = m_A \vec{v}_A + m_B \vec{v}_B + \ldots$

Elastic
- Moment is conserved
- Kinetic Energy is conserved
- Total Energy is conserved

Inelastic:
- Moment is conserved
- Kinetic Energy is not conserved
- Total Energy is conserved, but some KE is converted into a different form of energy (potential, internal, ...)

Problem Solving Strategy:

Is collision inelastic? $\xrightarrow{\text{No}}$ Is it elastic and 1D $\xrightarrow{\text{Not 1D}}$ $\sum \vec{p}_i = \sum \vec{p}_f$, $\sum E_{ki} = \sum E_{kf}$

$\downarrow$ Yes

ONLY moment conserved:

$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$

If completely inelastic:

$m_1 v_{1i} + m_2 v_{2i} = (m_1 + m_2) v_f$

$\downarrow$ Yes and 1D

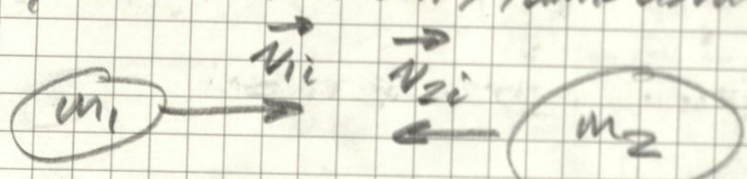
$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$

$v_{1i} + v_{1f} = v_{2i} + v_{2f}$

(b/c K is conserved)

Everything else (can't tell, don't know):
only conserve moment, think about energy later

Ex:



Find their velocity if they stick together:

$m_1 = 3\,kg$, $v_{1i} = 4\,m/s$ $m_2 = 5\,kg$, $v_{2i} = -4\,m/s$

Completely Inelastic!

$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f} = (m_1 + m_2) v_f$

$\Rightarrow v_f = \frac{m_1 v_{1i} + m_2 v_{2i}}{m_1 + m_2} = \frac{-8\,kg\,m/s}{3kg + 5kg} = -1\,m/s$

the two balls move together to the left

Find their velocity if they bounce off one another!

$m_1 = 3\,kg,\ v_{1i} = 4\,m/s \quad m_2 = 5\,kg,\ v_{2i} = -4\,m/s$

Elastic Collision!

$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$ Shortcut!

$v_{1i} + v_{1f} = v_{2i} + v_{2f}$

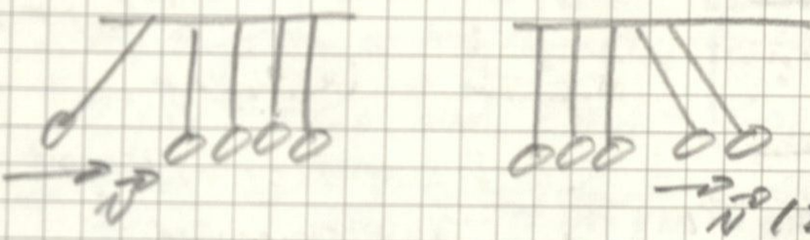
$\Rightarrow 3\,kg \cdot 4\,m/s - 5\,kg \cdot 4\,m/s = 3\,kg\, v_{1f} + 5\,kg\, v_{2f}$

$\Rightarrow 4\,m/s + v_{1f} = -4\,m/s + v_{2f}$ solve set of linear equations

$v_{1f} = -6\,m/s \quad v_{2f} = 2\,m/s$

3kg ball moves to left and 5kg ball moves to right

Can one ball set two balls in motion?



$p_i = mv$

$K_i = \frac{1}{2} m v^2$

$p_f = 2m \cdot \frac{v}{2} = mv$

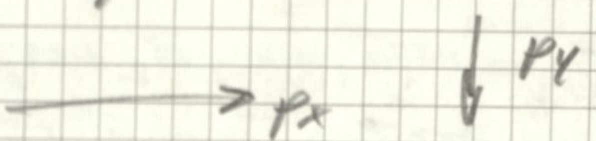
$K_f = \frac{1}{2} m \left(\frac{v}{2}\right)^2 + \frac{1}{2} m \left(\frac{v}{2}\right)^2 = \frac{1}{4} m v^2$

$p_i = p_f \rightarrow \Sigma \vec{p}$ is conserved

$K_i \neq K_f \rightarrow \Sigma K$ is not conserved

so NO!

EX - Suppose rain falls vertically on a rolling cart traveling horizontally. As a result of accumulating water, the KE?



$p_{xi} = m_c v_c \quad p_{xf} = (m_c + m_r) v_{c+r}$

$p_{xi} = p_{xf} \Rightarrow v_{c+r} = \frac{m_c v_c}{m_c + m_r}$

$K_{ci} = \frac{1}{2} m_c v_c^2 \Rightarrow K_{c+r,f} = \frac{1}{2}(m_c + m_r) v_{c+r}^2$

$\Rightarrow \frac{1}{2} \underbrace{\frac{m_c^2}{m_c + m_r}}_{< m_c} v_c^2 < K_{ci}$

Reading for Class #35:

5.20.24

8.5: Center of Mass: Mass-weighted average

$$x_{cm} = \frac{m_1x_1 + m_2x_2 + m_3x_3 + \dots}{m_1 + m_2 + m_3 + \dots} = \frac{\sum_i m_i x_i}{\sum_i m_i}$$

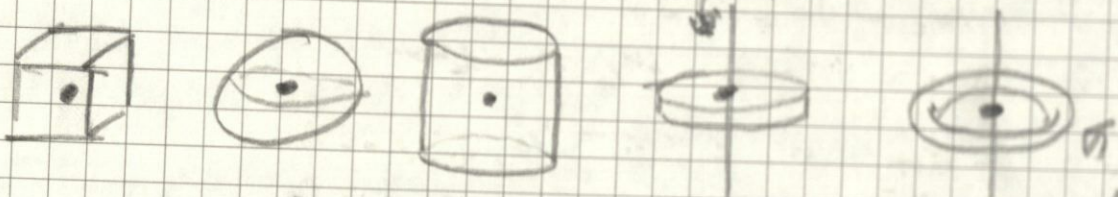
$$y_{cm} = \frac{\sum_i m_i y_i}{\sum_i m_i} \quad \dots$$

$$\vec{r}_{cm} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + m_3\vec{r}_3 + \dots}{m_1 + m_2 + m_3 + \dots} = \frac{\sum_i m_i \vec{r}_i}{\sum_i m_i}$$

center of mass of a systems of particles

masses of individual particles

Axis of symmetry



Donut

$$\vec{v}_{cm} = \frac{m_1\vec{v}_1 + m_2\vec{v}_2 + m_3\vec{v}_3 + \dots}{m_1 + m_2 + m_3 + \dots}$$

$M = m_1 + m_2 + \dots$

$\vec{P} = \vec{p}_1 + \vec{p}_2 + \dots$

momenta of individual particles

$\star \quad \vec{P} = M\vec{v}_{cm} = m_1\vec{v}_1 + m_2\vec{v}_2 + m_3\vec{v}_3$

If $\sum F_{ext} = 0$, $\vec{P}$ and $v_{cm} = \frac{\vec{P}}{M}$ are constant

External Forces and CM Motion: $\sum F_{ext} \neq 0$

$$\vec{a}_{cm} = \frac{d\vec{v}_{cm}}{dt} \Rightarrow M\vec{a}_{cm} = m_1\vec{a}_1 + m_2\vec{a}_2 + m_3\vec{a}_3 + \dots$$

Newton's 3rd Law

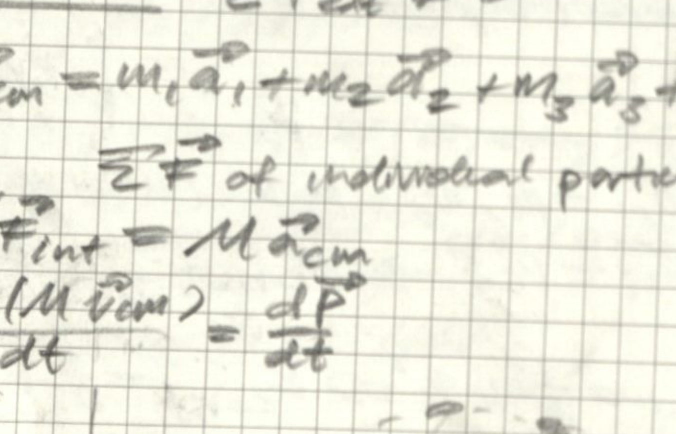
$\sum \vec{F}$ of individual particles

$$\sum \vec{F} = \sum \vec{F}_{ext} + \cancel{\sum \vec{F}_{int}}^{\,0} = M\vec{a}_{cm}$$

$$M\vec{a}_{cm} = M\frac{d\vec{v}_{cm}}{dt} = \frac{d(M\vec{v}_{cm})}{dt} = \frac{d\vec{P}}{dt}$$

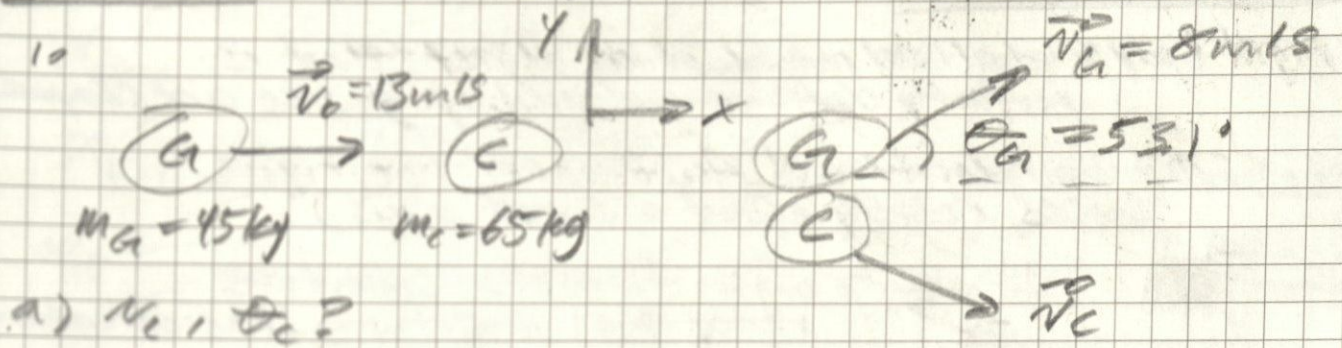
$$\sum \vec{F}_{ext} = M\vec{a}_{cm} = \frac{d\vec{P}}{dt}$$

(after a shell explodes, the fragments CM continues to follow the initial path)



Lecture Notes: More Collisions and CM 3.21.24

1.



a) v_C, θ_C?

Conservation of $\vec{p}$: $\sum \vec{p}_i = \sum \vec{p}_f$

x: $m_G v_0 = m_G v_G \cos\theta_G + m_C v_C \cos\theta_C$

y: $0 = m_G v_G \sin\theta_G - m_C v_C \sin\theta_C$

To find θ_C:

$m_C v_C \sin\theta_C = m_G v_G \sin\theta_G$

$m_C v_C \cos\theta_C = m_G v_0 - m_G v_G \cos\theta_G$

$\sum p_{xi} = \sum p_{xf}$ and $\sum p_{yi} = \sum p_{yf}$, so:

$* \ \frac{\sum p_{xi}}{\sum p_{yi}} = \frac{\sum p_{xf}}{\sum p_{yf}} \Rightarrow \tan\theta_C = \frac{v_G \sin\theta_G}{v_0 - v_G \cos\theta_G} \Rightarrow \theta_C = 38°$

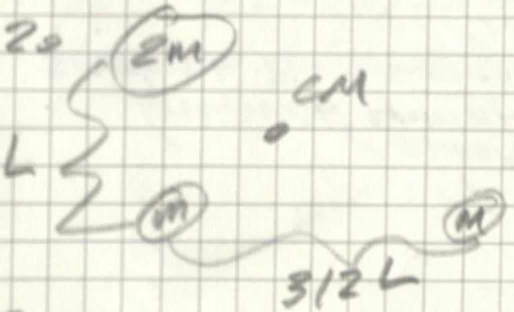
To find v_C:

use $m_C v_C \cos\theta_C = m_G v_0 - m_G v_G \cos\theta_G \Rightarrow v_C = 7.2 m/s$

b) ΔK?

$\Delta K = K_f - K_i = \frac{1}{2} m_C v_C^2 + \frac{1}{2} m_G v_G^2 - \frac{1}{2} m_G v_0^2 = -618 J$

2.

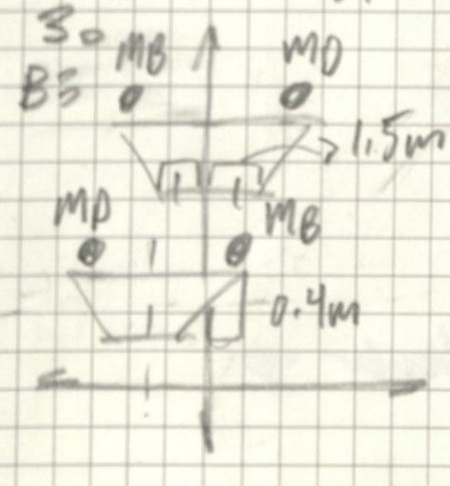


$$\vec{r}_{cm} = \frac{\sum m_i \vec{r}_i}{\sum m_i}$$

$$= \frac{m \cdot 0 + m \cdot \frac{3}{2} L \hat{i} + 2m L \hat{j}}{m + m + 2m}$$

$$= \frac{3}{8} L \hat{i} + \frac{1}{2} L \hat{j}$$

3.



$F_{ext} = 0 \Rightarrow \frac{d\vec{p}_{cm}}{dt} = 0$

$\vec{p}_{cm} = M_{tot} \vec{v}_{cm}$ = conserved

$\vec{v}_{cmi} = \vec{v}_{cmf} \Rightarrow 0 = 0$

$\therefore \vec{v}_{cm} = \frac{d\vec{r}}{dt} = 0 \rightarrow \vec{r}_{cmi} = \vec{r}_{cmf}$

$\Rightarrow x_{cmi} = x_{cmf}$

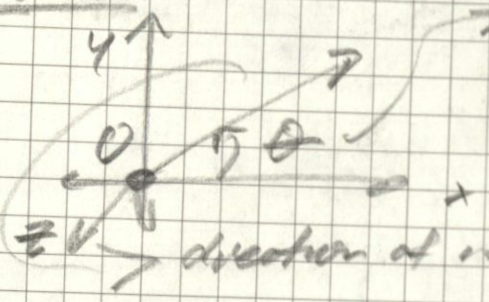
Reading for Class #26

3.21.24

rigid body: idealized model of an object that is perfectly definite and unchanging size and shape

9.1: Angular Velocity and Acceleration (of rigid bodies rotating about a fixed axis)

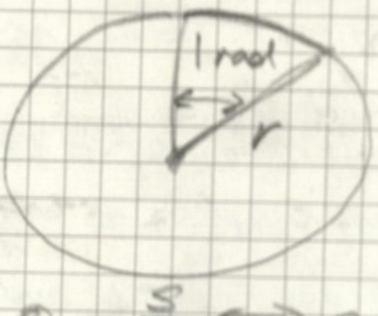
Position:



→ angle from +x specifies rotational position (has direction)

direction of rotation

SI: radians! → arc length $s = r$



Velocity:

$$\omega_{av-z} = \frac{\theta_2 - \theta_1}{t_2 - t_1} = \frac{\Delta\theta}{\Delta t}$$

$$\omega_z = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt}$$

$\theta = \frac{s}{r} \Leftrightarrow s = r\theta$

rotating about z-axis w/ direction * pure numbers

counterclockwise → $\Delta\theta > 0 \Rightarrow \omega_z > 0$

clockwise → $\Delta\theta < 0 \Rightarrow \omega_z < 0$

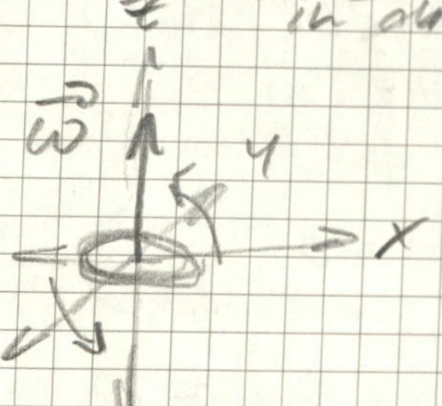
ω: angular speed, w/o direction

SI: 1 rev/s = 2π rad/s

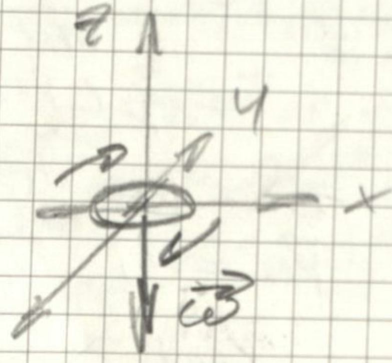
1 rev/m = 1 rpm = $\frac{2\pi}{60}$ rad/s

1 rad/s ~ 10 rpm

Angular Vectors: use right-hand rule with fingers curled around z and thumb pointing in direction of vector



$\omega_z > 0$



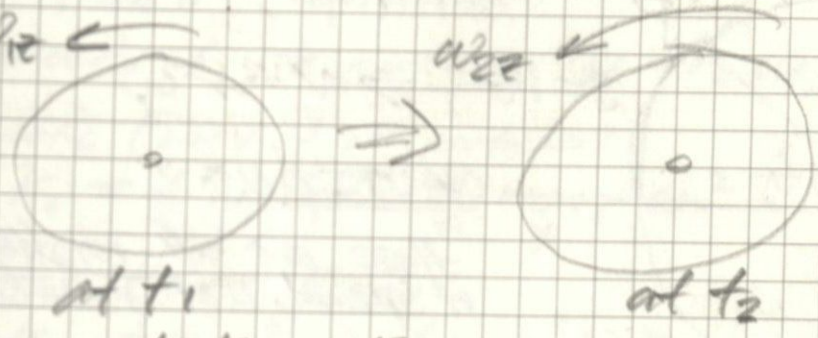
$\omega_z < 0$

CAUTION: angular vector is perpendicular to plane of rotation

Acceleration:

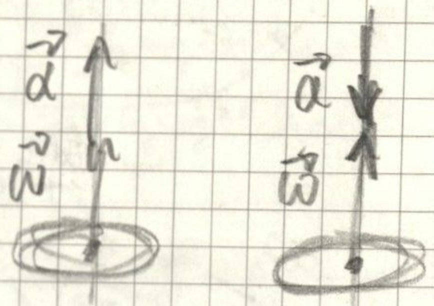
$$\alpha_{av-z} = \frac{\omega_{2z} - \omega_{1z}}{t_2 - t_1} = \frac{\Delta \omega_z}{\Delta t}$$

$$\alpha_z = \lim_{\Delta t \to 0} \frac{\Delta \omega_z}{\Delta t} = \frac{d\omega_z}{dt}$$



$\vec{\alpha}$ and $\vec{\omega}$ in same directions: Rotation speeding up

$\vec{\alpha}$ and $\vec{\omega}$ in opposite directions: Rotation slowing down



$$\alpha_z = \frac{d}{dt}\frac{d\theta}{dt} = \frac{d^2\theta}{dt^2} \quad \text{SI: rad/s}^2$$

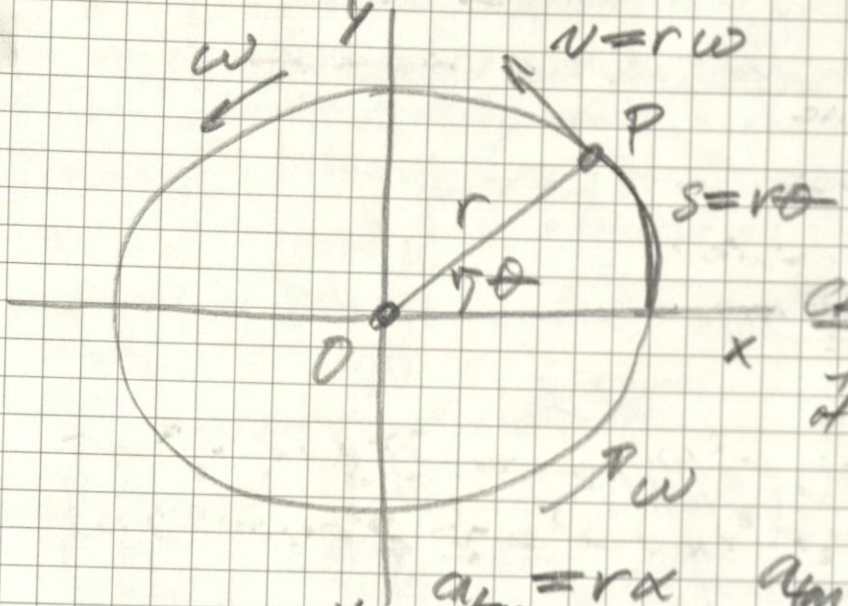
if $\alpha_z > 0$, ω_z increasing
if $\alpha_z < 0$, ω_z decreasing

9.2: Rotation with Constant Angular Acceleration

Straight-Line Motion with Constant Linear Acceleration	Fixed-Axis Rotation with Constant Angular Acceleration
$a_x = \text{constant}$	$\alpha_z = \text{constant}$
$v_x = v_{0x} + a_x t$	$\omega_z = \omega_{0z} + \alpha_z t$
$x = x_0 + v_{0x} t + \frac{1}{2} a_x t^2$	$\theta = \theta_0 + \omega_{0z} t + \frac{1}{2}\alpha_z t^2$
$v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$	$\omega_z^2 = \omega_{0z}^2 + 2\alpha_z(\theta - \theta_0)$
$x - x_0 = \frac{1}{2}(v_{0x} + v_x)t$	$\theta - \theta_0 = \frac{1}{2}(\omega_{0z} + \omega_z)t$

Reading for Class #27: 3.24.24

9.3: Relating Linear and Angular Kinematics



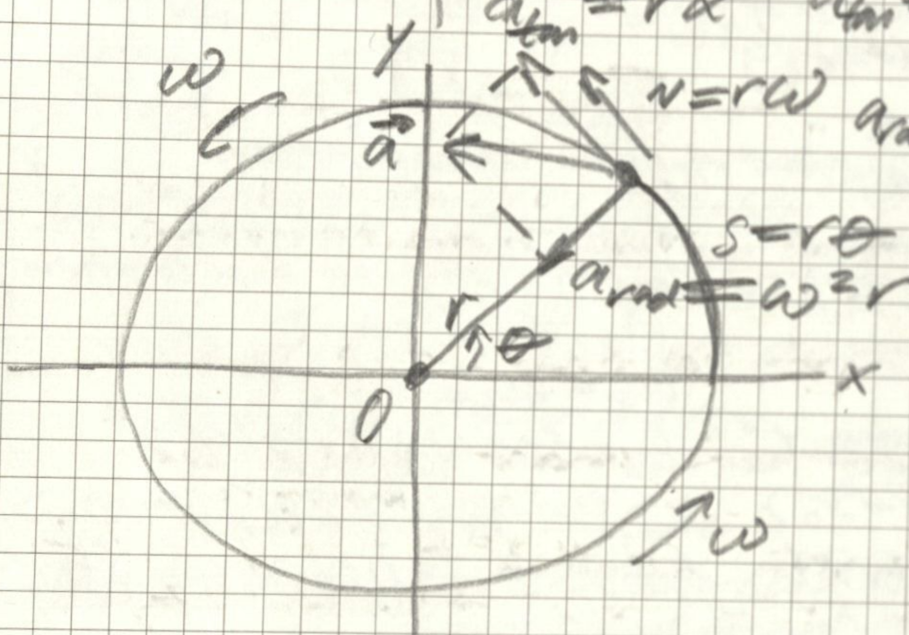
$$\left|\frac{ds}{dt}\right| = r\left|\frac{d\theta}{dt}\right|$$

$s = r\theta \Rightarrow v = r\omega$

CAUTION:

these are all relationships of speed, not velocity

$a_{tan} = r\alpha$ $\quad a_{tan} = \frac{dv}{dt} = r\frac{d\omega}{dt} = r\alpha$



$a_{rad} = \frac{v^2}{r} = \omega^2 r$

CAUTION:

use all angles in radians

9.4: Energy in Rotational Motion

Rigid body made up of particles $m_1, m_2, \ldots$ at distances $r_1, r_2, \ldots$ from axis of rotation.

m_i and r_i is perpendicular distance from AoR for any ith particle

velocity of ith particle: $v_i = r_i \cdot \omega$

Kinetic Energy of ith particle:

$$K_i = \frac{1}{2} m_i v_i^2 = \frac{1}{2} m_i r_i^2 \omega^2$$

Total Kinetic Energy:

$$K = \frac{1}{2} m_1 r_1^2 \omega^2 + \frac{1}{2} m_2 r_2^2 \omega^2 + \ldots = \sum_i \frac{1}{2} m_i r_i^2 \omega^2$$

$$\Rightarrow K = \frac{1}{2}(m_1 r_1^2 + m_2 r_2^2 + \ldots)\omega^2 = \frac{1}{2}\left(\sum_i m_i r_i^2\right)\omega^2$$

Moment of Inertia:

$$I = m_1 r_1^2 + m_2 r_2^2 + \ldots = \sum_i m_i r_i^2 \qquad \underline{SI}: kg \cdot m^2$$

"Moment referring to how a body's mass is distributed in space; nothing to do with a "moment" in time"

Rotational Kinetic Energy:

$$K = \frac{1}{2} I \omega^2 \qquad \underline{SI}: \omega \to rad,\ K \to J$$

"greater I → greater K, harder to get a body with more moment of inertia to start moving"

- mass close to axis → small I → easy to rotate
- mass far from axis → large I → hard to rotate

<u>CAUTION</u>: I depends on AoR chosen

not enough to say "$I = x\ kg \cdot m^2$"

have to be specific! "$I = x\ kg \cdot m^2$ about axis through A and B

common moments of inertia on formula sheet

Gravitational Potential Energy of Extended Bodies:

$$U = Mg y_{cm}$$

$$U = m_1 g y_1 + m_2 g y_2 + \ldots = (m_1 y_1 + m_2 y_2 + \ldots) g$$

$$m_1 y_1 + m_2 y_2 + \ldots = (m_1 + m_2 + \ldots) y_{cm} = M y_{cm}$$

Lecture Notes: 3.25.24

Linear: Rotational:

$x \longrightarrow \theta$

$v \longrightarrow \omega$

$a \longrightarrow \alpha$

$m \longrightarrow I$

Discrete:

$$I = \sum m_i r_i^2$$

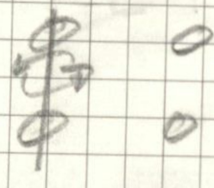
Continuous:

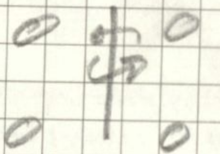
$$I = \int r^2 dm = \int r^2 \rho \, dV$$

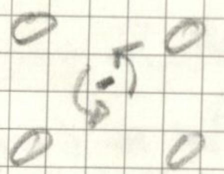
radius, mass integral, density, volume integral

Ex:

Which configuration has the largest K? given constant ω, m, and L

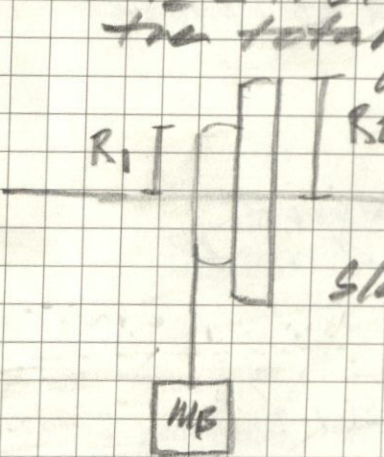
A  $I = m(0)^2 + m(0)^2 + ml^2 + ml^2 = 2ml^2$

B  $I = 4\left[m\left(\frac{l}{2}\right)^2\right] = \frac{4ml^2}{4} = ml^2$

C  $r = \frac{\sqrt{2}}{2} l$, $I = 4\left[m\left(\frac{\sqrt{2}}{2} l\right)^2\right] = 2ml^2$

A and C

Ex: Two metal disks $R_1 = 2.5\,cm$, $M_1 = 0.8\,kg$, $R_2 = 5\,cm$, $M_2 = 1.6\,kg$, are welded together. What is the total moment of inertia?



a) $I_{tot} = I_1 + I_2 = \frac{1}{2} M_1 R_1^2 + \frac{1}{2} M_2 R_2^2$

$= 2.25 \cdot 10^{-3}\,kg \cdot m^2$

$I_{disk} = \frac{1}{2} M R^2$

b/c) $K_i + U_i = K_f + \cancel{U_f}^{\,0}$, $v_B = R\omega \Rightarrow \omega = \frac{v_B}{R}$

$\Rightarrow \frac{1}{2} m_B v_B^2 + \frac{1}{2} I \omega^2 = m_B g h$

$\Rightarrow \frac{1}{2} m_B v_B^2 + \frac{1}{2} I \left(\frac{v_B}{R}\right) = m_B g h \Rightarrow v_B = \checkmark$

b/c) $m_B = 1.5\,kg$ block is suspended by a string wrapped around disk 1. If it is released from rest at 2m above the floor, what is its speed just before hitting the floor? What if it was attached to disk 2?

Reading for Class #28 : 3.26.24

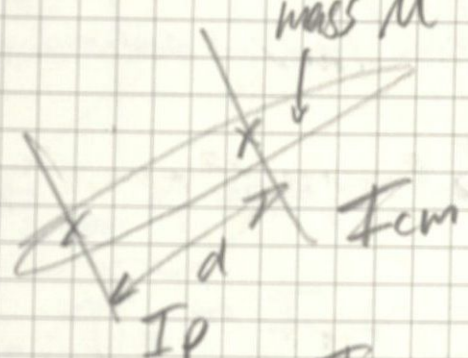
9.5 : Parallel-Axis Theorem

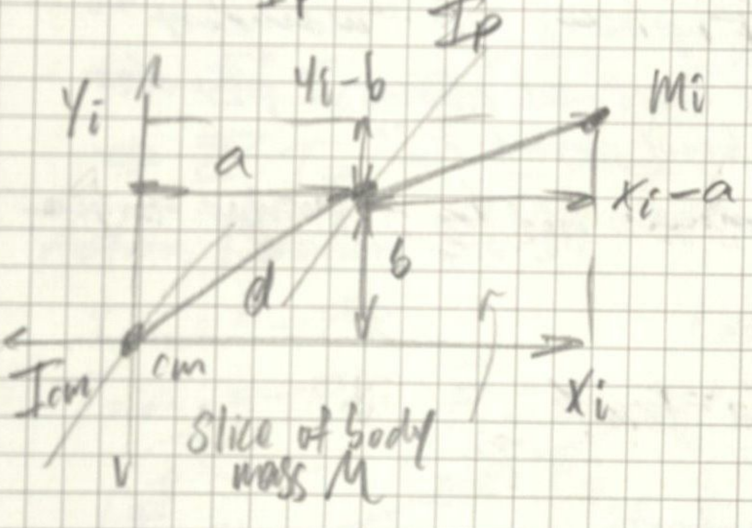
There are actually infinitely many moments of inertia

Parallel Axis Theorem relates moments of inertia to others parallel to the moment of inertia through the body's center of mass :

$$I_p = I_{cm} + Md^2$$

distance between two parallel axis





$$I_{cm} = \sum_i m_i (x_i^2 + y_i^2)$$

$$I_p = \sum_i m_i \left[(x_i - a)^2 + (y_i - b)^2 \right]$$

$$I_p = \underbrace{\sum_i m_i (x_i^2 + y_i^2)}_{I_{cm}} \underbrace{- 2a \sum_i m_i x_i - 2b \sum m_i y_i}_{x_{cm} - y_{cm}} + \underbrace{(a^2 + b^2) \sum_i M_i}_{Md^2}$$

$\Rightarrow 0 - 0 = 0$

b/c at origin

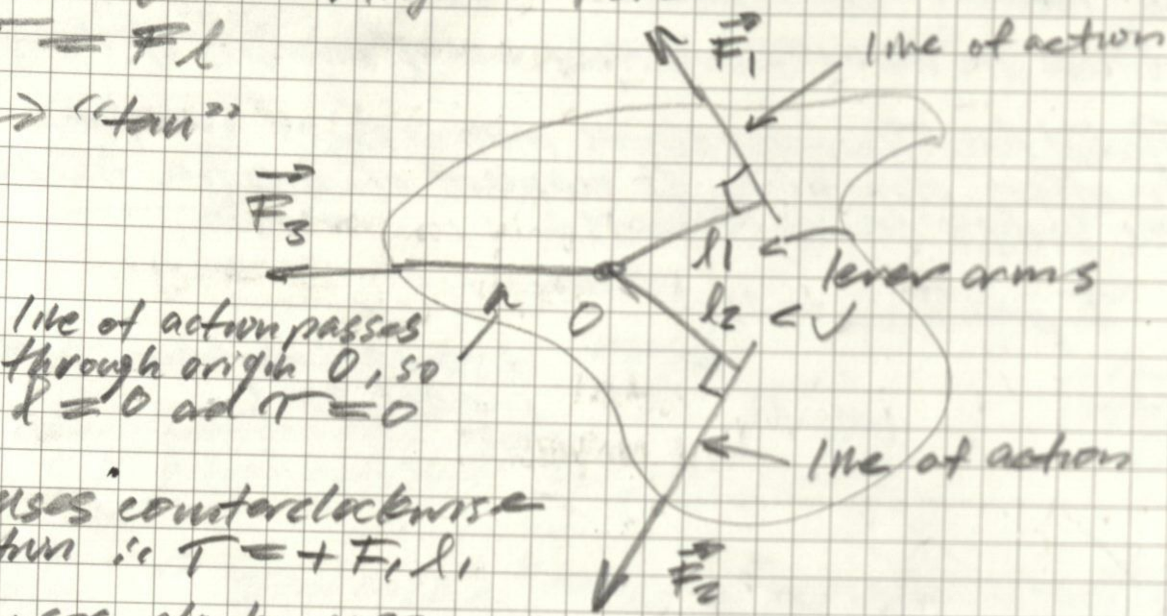
Reading for Class #24:

4.2.24

10.1: Torque — Angular Force

$\tau = Fl$

↳ "tan"



$\vec{F}_1$ causes counterclockwise rotation ∴ $\tau = +F_1 l_1$

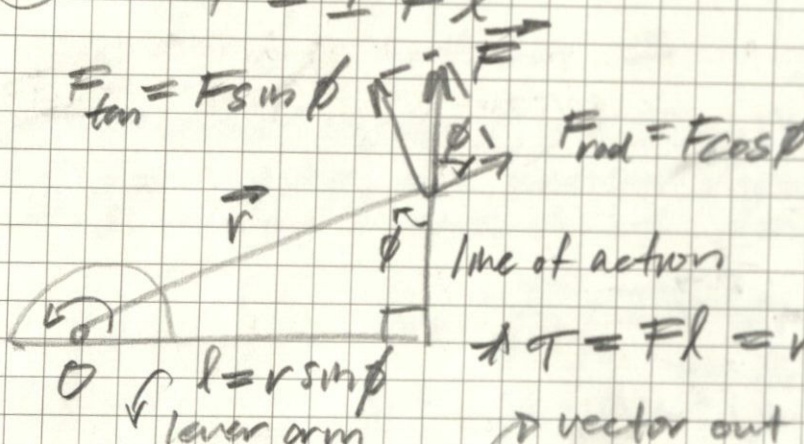
$\vec{F}_2$ causes clockwise rotation ∴ $\tau = -F_2 l_2$

Physicists → "torque"; Engineers → "moment"

CAUTION: torque is always measured about a point
"torque of $\vec{F}$ about point x"

(+↺) symbol is used to indicate choice of positive torque

$\tau = \pm Fl$



∧ $\tau = Fl = rF\sin\phi = F_{tan} r$

↗ vector out of page

•

× ↳ vector into page

Torque Vectors:

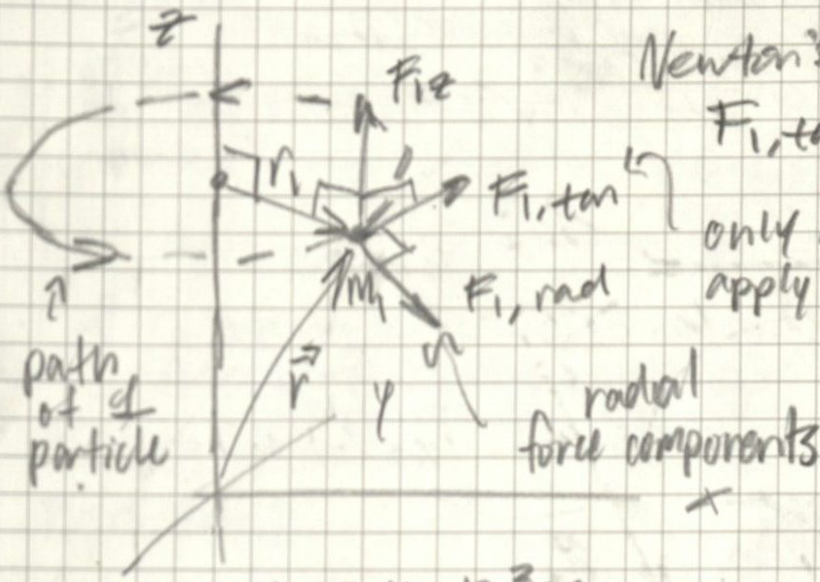
$\vec{\tau} = \vec{r} \times \vec{F}$

↳ vector from O to where $\vec{F}$ acts

∧ in accordance to right-hand rule

— if you point your fingers in your RH in the direction of $\vec{r}$ and curl them in the direction of $\vec{F}$, your thumb points in $\vec{\tau}$

10.2 : Torque and Angular Acceleration for a Rigid Body



Newton's Law:

$$F_{1,tan} = m_1 a_{1,tan}$$

only component applying a z-torque

$$F_{1,tan} r_1 = m_1 r_1^2 \alpha_z$$

$$\tau_{1z} = I_1 \alpha_z = m_1 r_1^2 \alpha_z$$

$$\Rightarrow \sum \tau_{iz} = \left(\sum m_i r_i^2\right) \alpha_z$$

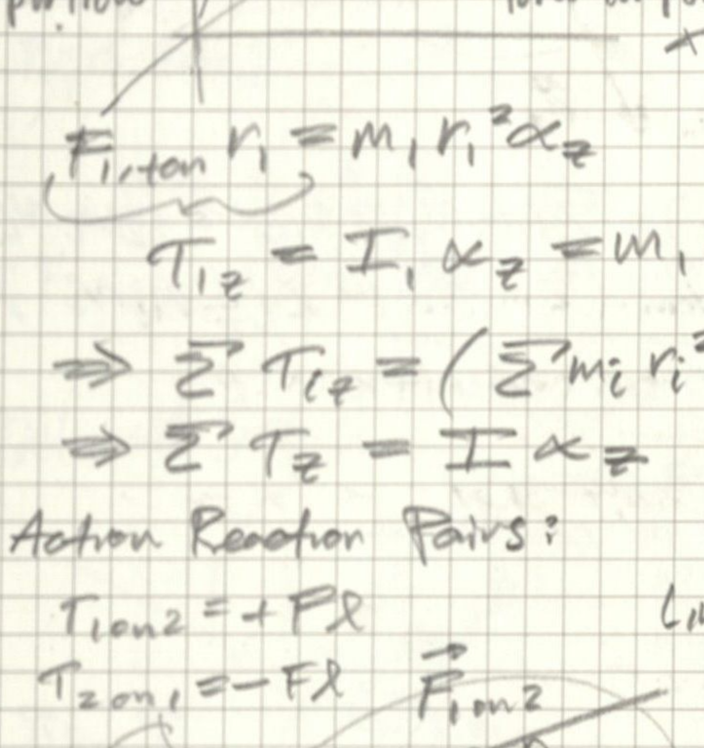
$$\Rightarrow \sum \tau_z = I \alpha_z$$

Action Reaction Pairs:

$\tau_{1 on 2} = +Fl$

$\tau_{2 on 1} = -Fl$

Line of action of both particles



* assume all weight is concentrated at center of mass of body

Reading for Class #30:

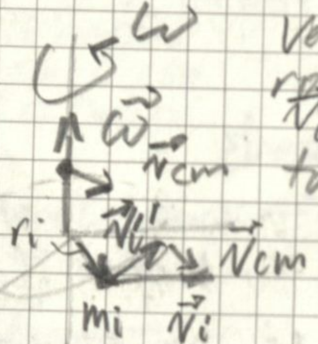
4.5.24

10.3: Rigid-Body Rotation About a Moving Axis
- combined translation and rotation
  - translation of the COM
  - rotation about the axis of COM

Energy Relationships:

$$K = \frac{1}{2}Mv_{cm}^2 + \frac{1}{2}I_{cm}\omega^2$$

most of the time: $K_{cm} + K_r$



Velocity $\vec{v}_i$ of particle rotating, translating; $\vec{v}_{cm}$ and $\vec{v}_i'$ relative to COM

$$K = \sum K_i = \sum\left(\tfrac{1}{2}m_i v_{cm}^2\right) + \sum\left(m_i \vec{v}_{cm}\cdot\vec{v}_i'\right) + \sum\left(\tfrac{1}{2}m_i v_i'^2\right)$$

$$K = \frac{1}{2}\left(\sum m_i\right)v_{cm}^2 + \underbrace{\vec{v}_{cm}\cdot\left(\sum m_i\vec{v}_i'\right)}_{0 \text{ by definition}} + \sum\left(\tfrac{1}{2}m_i v_i'^2\right)$$

Rolling Without Slipping:

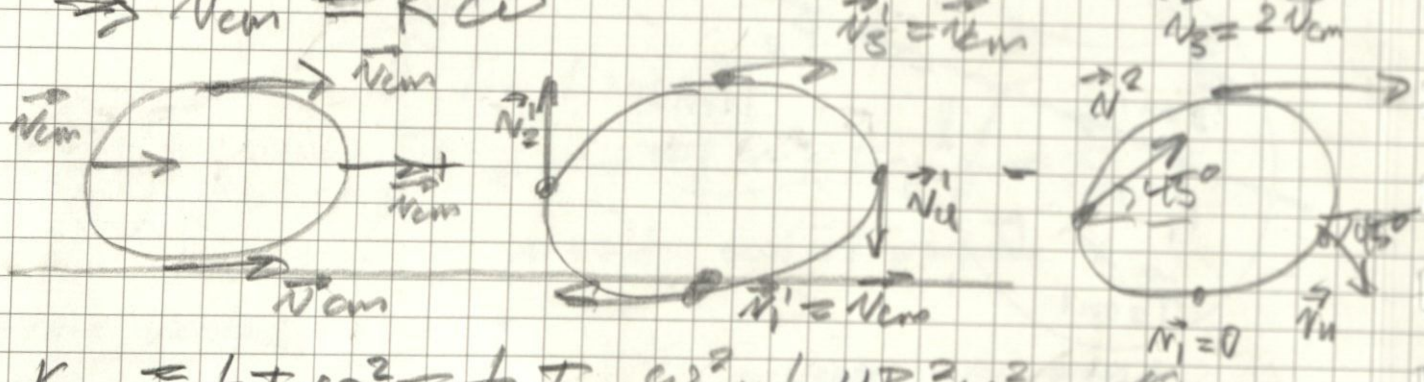
$\vec{v}_1' = -\vec{v}_{cm}$ → COM velocity

↳ velocity of the point of contact

$v_1' = R\omega$

↳ wheel radius

$\Rightarrow v_{cm} = R\omega$



$$K_{wheel} = \frac{1}{2}I_1\omega^2 = \frac{1}{2}I_{cm}\omega^2 + \frac{1}{2}MR^2\omega^2$$
$$= \frac{1}{2}I_{cm}\omega^2 + \frac{1}{2}Mv_{cm}^2$$

* $v_{cm} = R\omega$ only if there is rolling without slipping

Combined Dynamics:

$$\sum \vec{F}_{ext} = M\vec{a}_{cm}$$

$$\sum \tau_z = I_{cm}\alpha_z$$

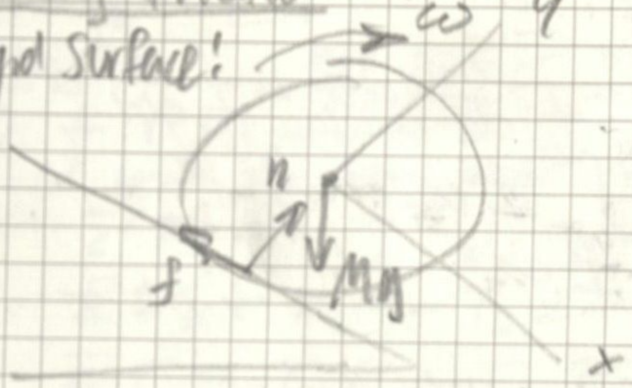
↳ assumes AoR is stationary, only use if:

1. axis through COM is an AoS
2. axis does not change direction

& is usually not at rest in an inertial frame of reference

Rolling Friction:

Rigid Surface:

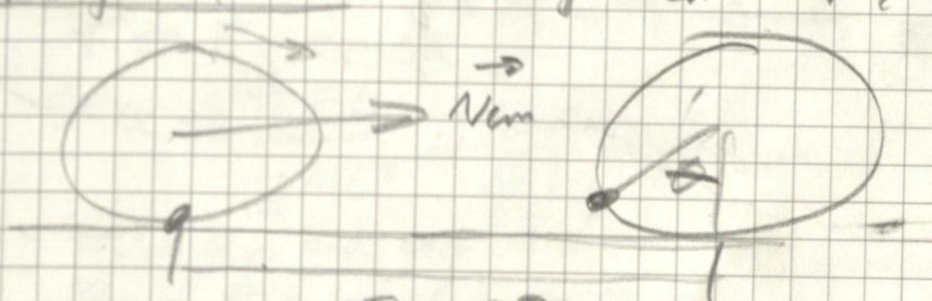


normal force produces no torque about the center of the sphere

Deformed Surface:

normal force produces a torque about the center of the sphere that opposes motion

Rolling Motions $\quad E_{rolling} = K_{cm} + K_r, \; E_{sliding} = K_{cm}$



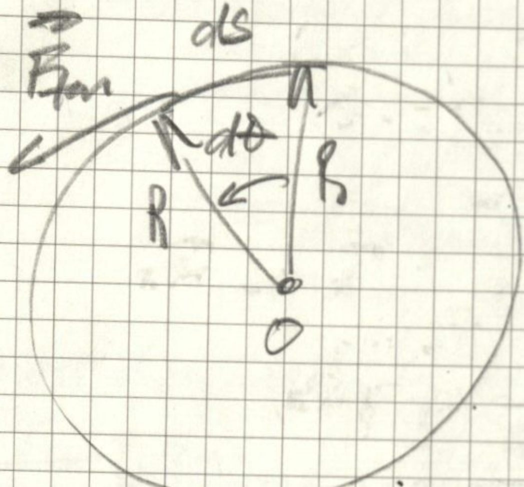
$$s = r\theta$$

$$v_{cm} = \frac{ds}{dt} = \frac{d}{dt}(r\theta) = r\frac{d\theta}{dt} = r\omega$$

$$a_{cm} = \frac{dv_{cm}}{dt} = \frac{d}{dt}(r\omega) = r\frac{d\omega}{dt} = r\alpha$$

Reading for Class #31:

10.4: Work and Power in Rotational Motion



$$dW = F_{tan} R\, d\theta$$

$$\Rightarrow dW = \tau_z\, d\theta$$

$$\Rightarrow W = \int_{\theta_1}^{\theta_2} \tau_z\, d\theta$$

$$\Rightarrow W = \tau(\theta_2 - \theta_1) = \tau_z \Delta\theta$$

Energy: $\tau_z d\theta = (I\alpha_z) d\theta = I \frac{d\omega_z}{dt} d\theta = I \frac{d\theta}{dt} d\omega_z$

$= I \omega_z d\omega_z \Leftrightarrow W_{tot} = \int_{\omega_1}^{\omega_2} I \omega_z\, d\omega_z$

$\Rightarrow W_{tot} = \frac{1}{2} I \omega_2^2 - \frac{1}{2} I \omega_1^2$

Power:

$\frac{dW}{dt} = \tau_z \frac{d\theta}{dt}$

$P = \tau_z \omega_z$

Analogs:

$W = \int_{\theta_1}^{\theta_2} \tau_z\, d\theta \longleftrightarrow W = \int_{x_1}^{x_2} F_x\, dx$

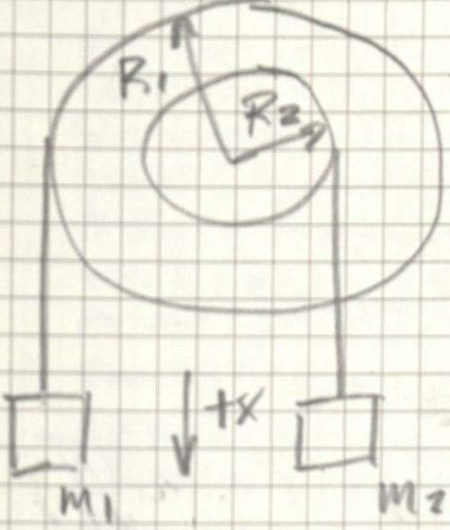
$W = \tau_z \Delta\theta \longleftrightarrow W = F s$

$W_{tot} = \frac{1}{2} I \omega_2^2 - \frac{1}{2} I \omega_1^2 \longleftrightarrow W_{tot} = \frac{1}{2} m v_2^2 - \frac{1}{2} m v_1^2$

$P = \tau_z \omega_z \longleftrightarrow P = \vec{F} \cdot \vec{v}$

Physics Tutorial Examples: 9.10.24

1.



a) find m_2 given m_1 s.t. $\alpha_z = 0$

$\sum F_1 = T_1 - m_1 g = 0$

$\sum F_2 = T_2 - m_1 g = 0$

$\sum \tau = \tau_1 - \tau_2$

$= F_{1,tan} R_1 - F_{2,tan} R_2$

$= T_1 R_1 - T_2 R_2$

$= m_1 g R_1 - m_2 g R_2$

$\Rightarrow m_2 = \frac{m_1 R_1}{R_2}$

b) if n kg added to m_1, find nonzero α with the same m_2

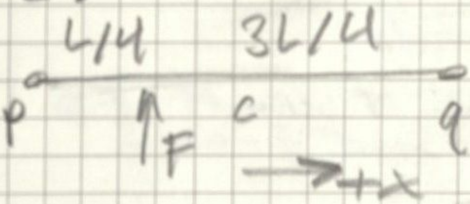
$\sum F_1 = m_1 g - T_1 = m_1 a_1 = m_1 R_1 \alpha$

$\sum F_2 = m_2 g - T_2 = -m_2 a_2 = -m_2 R_2 \alpha$

$\sum \tau = T_1 R_1 - T_2 R_2 = I \alpha$

$\Rightarrow \alpha = \frac{m_1 R_1 g - m_2 R_2 g}{I + m_1 R_1^2 + m_2 R_2^2}$

2.



a) find a_{cm}

$\sum F = M_{tot} a_{cm}$

$\Rightarrow a_{cm} = \frac{\sum F}{M_{tot}}$

b) Find α_c

$\tau = I\alpha = \frac{1}{12} ML^2 \alpha$

$\tau = Fr = F \frac{L}{4}$

$\Rightarrow \alpha = \frac{\frac{1}{4} FL}{\frac{1}{12} ML^2} = \frac{3F}{ML} = \frac{3}{L} a_{cm}$

c) Find a_p and a_q

$a_T = R\alpha = \frac{L}{2}\alpha = \frac{3}{2} a_{cm}$

$a_p = a_{cm} + \frac{3}{2} a_{cm} = \frac{5}{2} a_{cm}$

$a_q = a_{cm} - \frac{3}{2} a_{cm} = \frac{1}{2} a_{cm}$

Reading for Class #52:

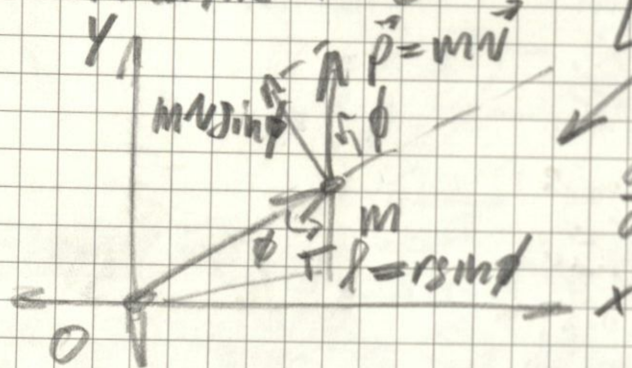
4.12.24

10.5: Angular Momentum

$\vec{\tau} = \vec{r} \times \vec{F}$ position vector relative to O

$\vec{L} = \vec{r} \times \vec{p} = \vec{r} \times m\vec{v}$ linear momentum

angular momentum relative to O



$L = mvr\sin\phi = mvl$

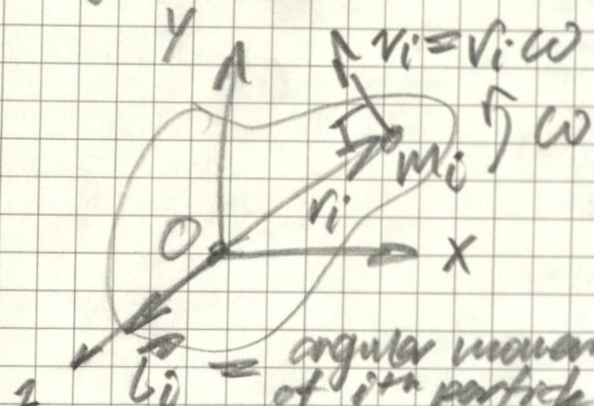
Note:

$$\frac{d\vec{L}}{dt} = \left(\frac{d\vec{r}}{dt} \times m\vec{v}\right) + \left(\vec{r} \times m\frac{d\vec{v}}{dt}\right)$$

$$= \underbrace{(\vec{v} \times m\vec{v})}_{\vec{0}} + \underbrace{(\vec{r} \times m\vec{a})}_{\vec{r} \times \vec{F}}$$

$$\Rightarrow \frac{d\vec{L}}{dt} = \vec{r} \times \vec{F} = \vec{\tau}$$

Rigid Bodies:



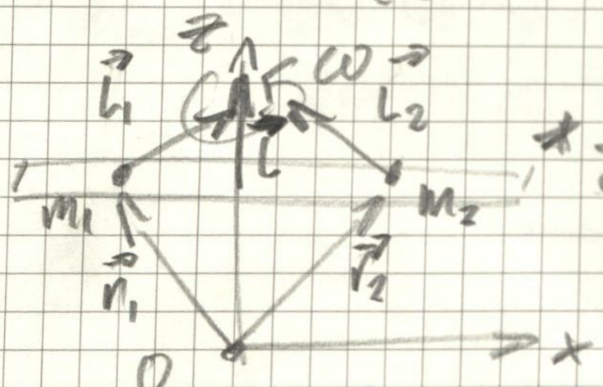
$L_i = m_i(r_i \omega) r_i = m_i r_i^2 \omega$

* each particle on a rotating rigid body has $\vec{r}_i$ + $\vec{v}_i$ magnitude

$\vec{L}_i$ = angular momentum of i^{th} particle of RB

I of slice of RB about z-axis

$$L = \sum L_i = \left(\sum m_i r_i^2\right)\omega = I\omega$$



$\vec{L} = I\vec{\omega}$

* so $\vec{L}$ is a scalar multiple of $\vec{\omega}$ & in same direction

$$\sum \vec{\tau} = \frac{d\vec{L}}{dt}, \quad \sum \tau_z = I\alpha_z$$

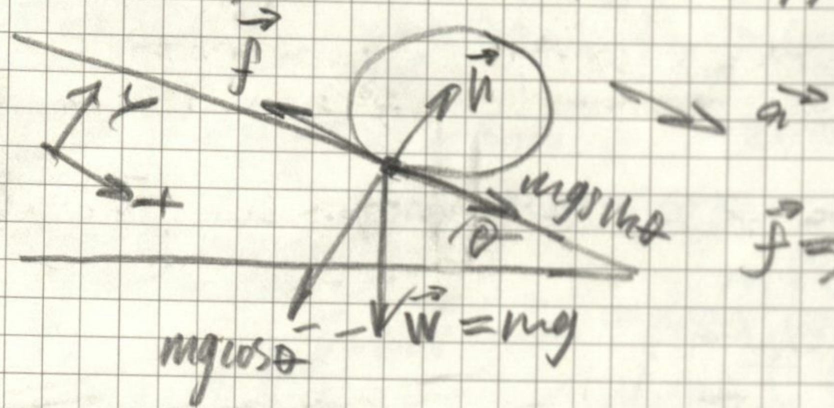
International System of Units (SI Units) 4.12.24

time (t): s
position ($\vec{r}$): m
velocity ($\vec{v}$): m/s
acceleration ($\vec{a}$): m/s^2
mass (m): kg
force ($\vec{F}$): N = kg · m/s^2
work (W): J = N · m
power (P): W = J/s
energy (K or U): J
momentum ($\vec{p}$): kg · m/s
impulse ($\vec{J}$): N · s = kg · m/s
angle (θ): rad = π/180 deg
angular velocity (ω): rad/s
angular acceleration (α): rad/s^2
moment of inertia (I): kg · m^2
torque ($\vec{\tau}$): N · m = kg · m^2/s^2
angular work (W): J
angular momentum ($\vec{L}$): kg · m^2/s

HW10 Ex:

11.12.24

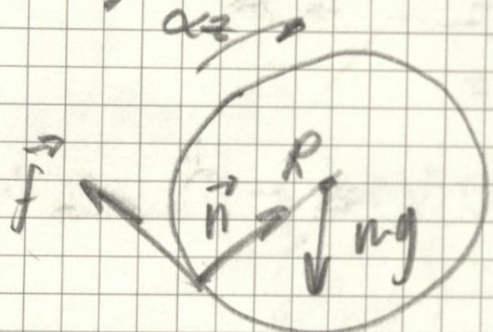
Minimum μ to roll without slipping



$\vec{f} = \mu\vec{n} = \mu mg\cos\theta$

$\Sigma\vec{F} = mg\sin\theta - \mu mg\cos\theta = ma$

$\Rightarrow g(\sin\theta - \mu\cos\theta) = ma$



$\Sigma\tau = \tau_f = I\alpha$

$\Rightarrow \mu_s mg\cos\theta R = \frac{2}{5}mR^2\alpha$

no slipping $\therefore \alpha = \frac{a}{R}$

$\Rightarrow \mu g\cos\theta = \frac{2}{5}a$

$\Rightarrow a = \frac{5}{7}g\sin\theta, \ \mu = \frac{2}{7}\tan\theta$

Reading for Class #33 4.14.24

10.6: Conservation of Angular Momentum

When net external torque acting on a system is 0, the total angular momentum of a system is conserved:

$$\sum \vec{\tau} = \frac{d\vec{L}}{dt} . \quad \sum \vec{\tau} = 0 \Leftrightarrow \frac{d\vec{L}}{dt} = 0 \Leftrightarrow \vec{L} \text{ constant}$$

$$\Rightarrow I_1 \omega_{1z} = I_2 \omega_{2z}$$

Ex.

Suppose A exerts $\vec{F}_{A on B}$ to B, with a torque of $\vec{\tau}_{A on B}$

$$\vec{\tau}_{A on B} = \frac{d\vec{L}_B}{dt}$$

At the same time B exerts $\vec{F}_{B on A}$ to A with a torque of $\vec{\tau}_{B on A}$

$$\vec{\tau}_{B on A} = \frac{d\vec{L}_A}{dt}$$

Since $\vec{F}_{A on B} = -\vec{F}_{B on A} \Leftrightarrow \vec{\tau}_{A on B} = -\vec{\tau}_{B on A}$:

$$\frac{d\vec{L}_A}{dt} = \frac{d\vec{L}_B}{dt} = 0$$

Since $\vec{L}_A + \vec{L}_B = \sum \vec{L}$ of system:

$$\frac{d\vec{L}}{dt} = 0$$

* torques of internal forces can transfer angular momentum from one object to another, but cannot change the total angular momentum of the system

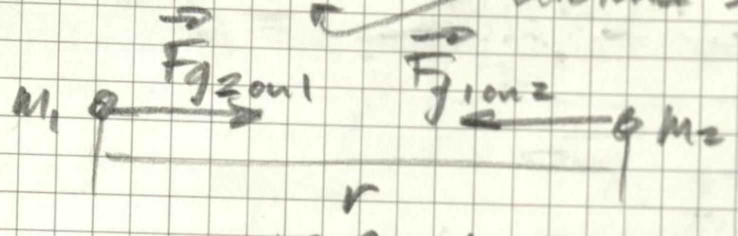
Reading for Class #35: 4.18.24

13.1: Newton's Laws of Gravitation

gravitational constant

$$F_g = \frac{G m_1 m_2}{r^2}$$

distance between m_1 and m_2



$$F_{g\,1on2} = F_{g\,2on1}$$

CAUTION: G and g are not the same

g: acceleration due to gravity (9.8 m/s on Earth)

G: universal gravitational force between any two objects in any space

Spherically Symmetric Objects:

mass of a spherically symmetric Earth

$$F_g = \frac{G m_E m}{r^2}$$

* F_g of spherically symmetric objects is the same as F_g of idealized particles
* points inside spherically symmetric objects do not have this same F_g expression

Gravitational Constant:

$$G = 6.67 \cdot 10^{-11}\, N \cdot m^2/kg^2$$

Since $N = 1 kg \cdot m/s^2$, SI $= m^3/kg \cdot s^2$

Superposition of Forces: gravitational forces combine vectorially

(if two masses exert a force on a third, $\vec{F_g}$ is a vector sum of those first two masses)

13.2 = Weight

gravitational constant

mass of the earth

mass of the object

$$w = F_g = \frac{G m_E m}{R_E^2}$$

radius of the earth

weight equals gravitational force the earth exerts on an object

$$g = \frac{G m_E}{R_E^2}$$

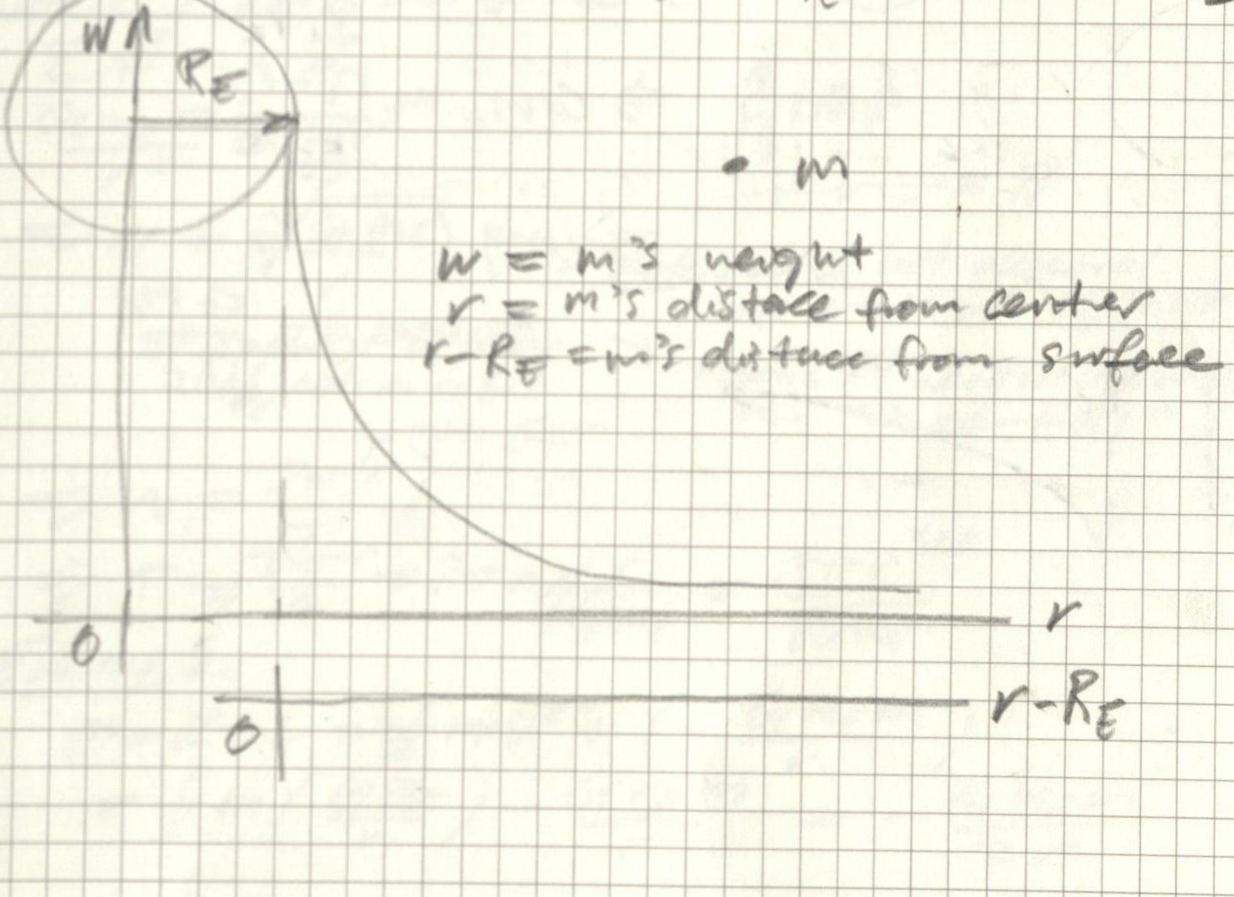
acceleration due to gravity at the earth's surface

$$m_E = \frac{g R_E^2}{G} = 5.96 \cdot 10^{24}\,\text{kg}$$

can measure mass of planets this way

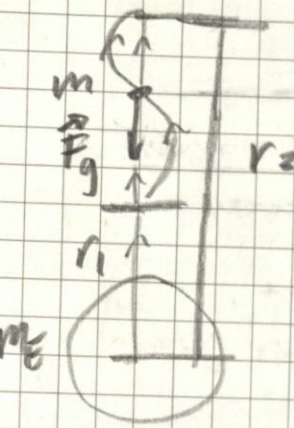
$$w = F_g = \frac{G m_E m}{r^2}$$

distance r from the center of the earth ($r - R_E$ above surface)



13.3: Gravitational Potential Energy 4.18.24

$U_g = mgy$ does not depend on an objects height



$$W_g = \int_{r_1}^{r_2} F_r \, dr$$

conservative force ∴ independent of path

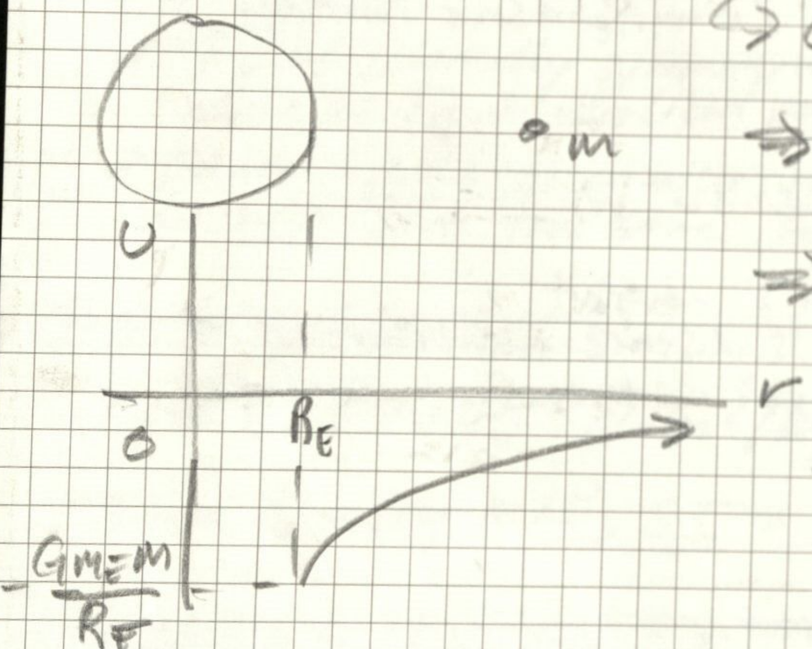
$$F_r = -\frac{G m_E m}{r^2}$$

radial component of F_g → outward from the center of the earth

F_g is normally an inward force

$$W_g = -G m_E m \int_{r_1}^{r_2} \frac{dr}{r^2} = \underbrace{\frac{G m_E m}{r_2} - \frac{G m_E m}{r_1}}$$

$$\therefore U_g = -\frac{G m_E m}{r}$$



$$\Rightarrow G m_E m \frac{r_1 - r_2}{r_1 r_2}$$

$$\Rightarrow G m_E m \frac{r_1 - r_2}{R_E^2}$$

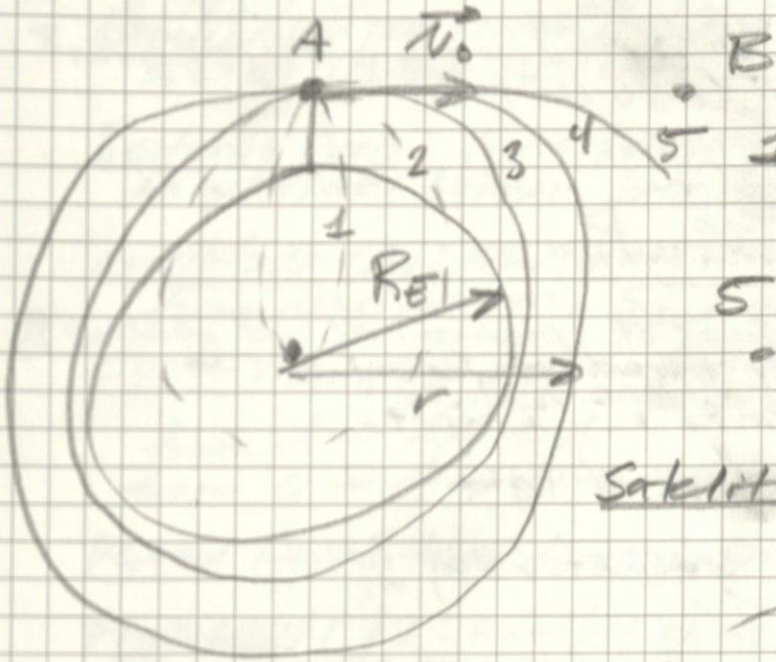
$$\Rightarrow mg(r_1 - r_2)$$

$$(\text{since } g = \frac{G m_E}{R_E^2}$$

$$\Rightarrow mgy$$

Reading for Class #36: 4.22.24

13.4: The Motion of Satellites

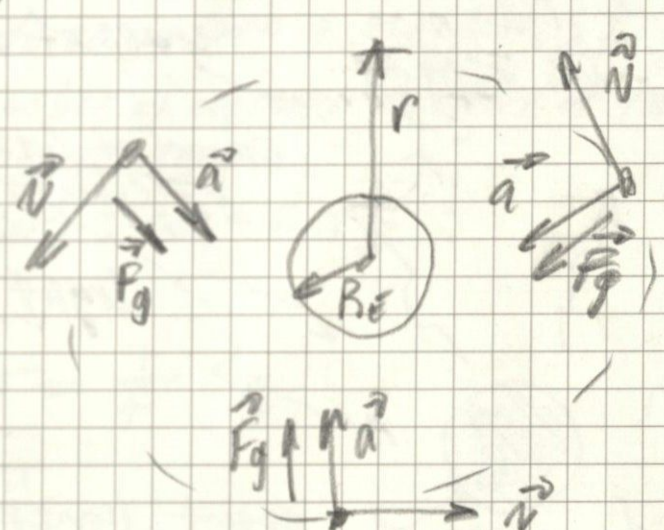


1-4: closed orbits
- ellipses, sometimes circles

5: open orbit:
- never return to starting position

Satellites: Circular Orbits

$\vec{a} \perp \vec{v}$ →
v constant



Newton's second law:

$$\sum \vec{F} = m\vec{a}$$

$$\frac{G m_E m}{r^2} = \frac{m v^2}{r}$$

$$\Rightarrow v = \sqrt{\frac{G m_E}{r}}$$ → note: does not depend on mass of object (state of weightlessness i.e. "free-fall")

↳ speed of circular object in orbit

$$\Rightarrow v = \frac{2\pi r}{T}$$

$$\Rightarrow T = \frac{2\pi r}{v} = 2\pi r\sqrt{\frac{r}{G m_E}} = \frac{2\pi r^{3/2}}{\sqrt{G m_E}}$$

Energy:

$$E = K + U = \frac{1}{2} m v^2 + \left(-\frac{G m_E m}{r}\right)$$

$$= \frac{1}{2} m\left(\frac{G m_E}{r}\right) - \frac{G m_E m}{r} = -\frac{G m_E m}{2r}$$

(all only for circular orbit)

13.8 = Black Holes

Escape Speed of a Star = escape speed

$$\rho = \frac{M}{V} = \frac{M}{\frac{4}{3}\pi R^3} = \frac{1.99 \cdot 10^{30} kg}{\frac{4}{3}\pi (6.96 \cdot 10^{8} m)^3} = 1410 kg/m^3$$

↳ average density of the sun

$$v = \sqrt{\frac{2GM}{R}} = \sqrt{\frac{8\pi G\rho}{3}}\, R$$

↳ escape speed for object of spherical M and radius R

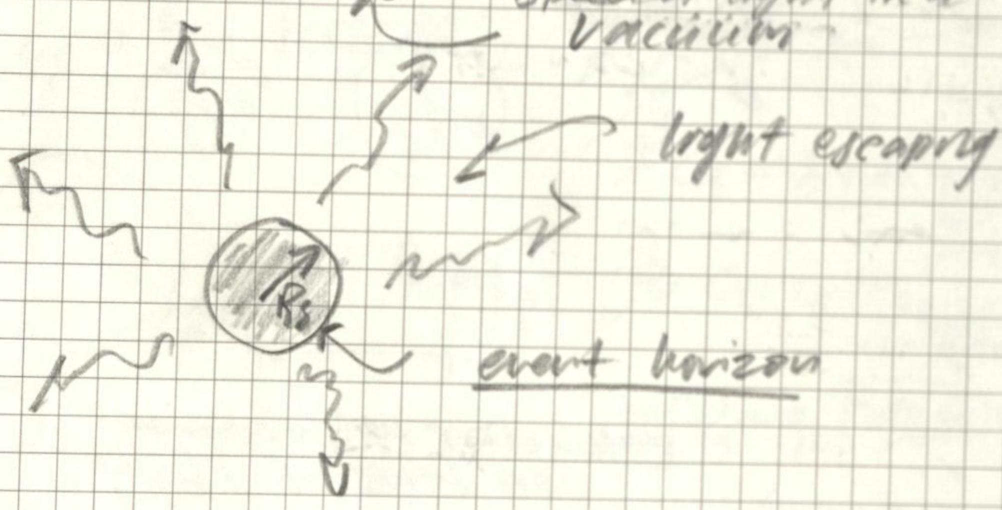
Schwarzschild Radius = escape radius

$$c = \sqrt{\frac{2GM}{R_S}}$$ ← speed of light

$$\Rightarrow R_S = \frac{2GM}{c^2}$$

mass of black hole

speed of light in a vacuum



Mass:

$$T = \frac{2\pi a^{3/2}}{\sqrt{G m_x}} \Rightarrow m_x = \frac{4\pi^2 a^3}{G T^2}$$

Reading for Class #37 4.23.24

14.1 : Describing Oscillation

Equilibrium position: stable (static) position of an oscillating system

Restoring Force: force bringing a displaced object in an oscillating system back to equilibrium

Amplitude (A): maximum magnitude of displacement from equilibrium, SI: m

Cycle: complete round-trip of oscillation

from $A \to -A \to A$

from $0 \to A \to 0 \to -A \to 0$

Period (T): time to complete a cycle, SI: s

Frequency (f): number of cycles in a unit of time

SI: Hz "hertz" = 1 cycle/s = 1 s^{-1}

Angular Frequency (ω): $\omega = 2\pi f$

SI: rad/s

$$f = \frac{1}{T} \Longleftrightarrow T = \frac{1}{f}$$

$$\omega = 2\pi f = \frac{2\pi}{T}$$

CAUTION: one period spans a complete cycle

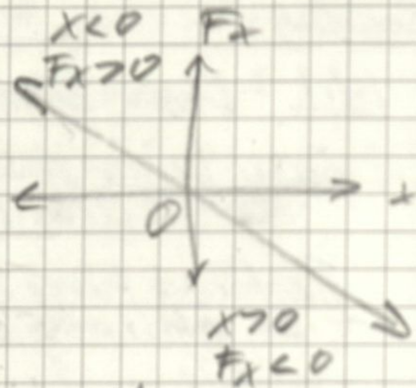
14.2: Simple Harmonic Motion

$F_x = -kx$

restoring force by an ideal spring

displacement

force constant of spring



$a_x = \frac{d^2x}{dt^2} = -\frac{k}{m}x$

not constant

acceleration and displacement have opposite signs

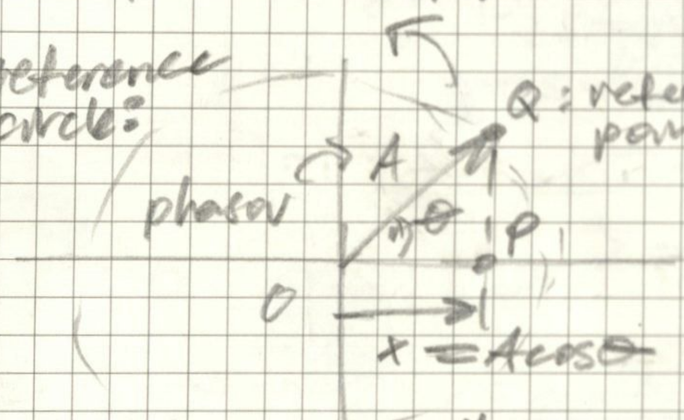
harmonic oscillator: object undergoing SHM

- SHM is an ideal approximation of periodic motion

Circular Motion and SHM Eqns:

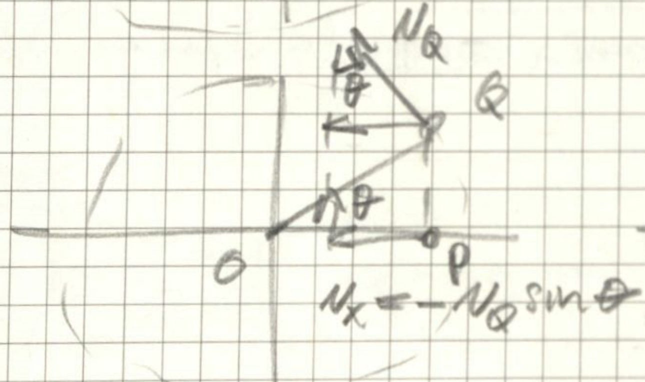
- SHM is a projection of uniform circular motion onto a diameter

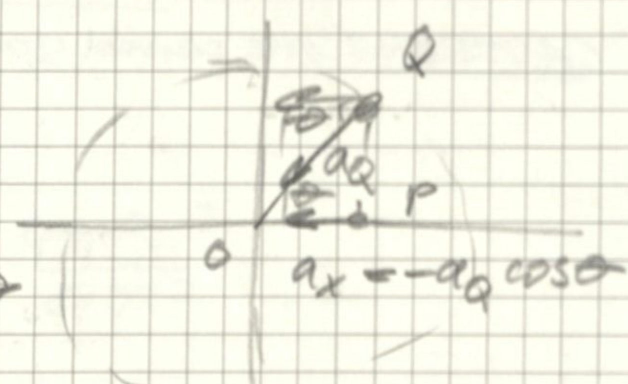
reference circle:



$x = A\cos\theta$

$a_Q = \omega^2 A$





$a_x = -a_Q\cos\theta = -\omega^2 A\cos\theta \iff a_x = -\omega^2 x$

$\omega^2 = \frac{k}{m} \iff \omega = \sqrt{\frac{k}{m}}$

force constant of restoring force

angular speed equals angular frequency

CAUTION: don't confuse ω and f

$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

$$T = \frac{1}{f} = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}}$$

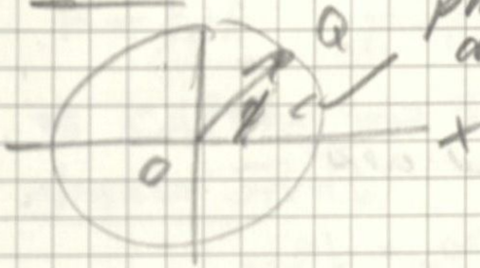
Period and Amplitude for SHM:

- in SHM, T and f do not depend on A

Displacement, Velocity, and Acceleration:

@ $t = 0$

phaser angle : $\theta = \omega t + \phi$



$$x = A\cos(\omega t + \phi)$$

$$\omega T = \sqrt{\frac{k}{m}}\,T = 2\pi \iff T = 2\pi\sqrt{\frac{m}{k}}$$

- increasing m increases T
- increasing k decreases T
- changing A has no effect on T

Consequently,

ϕ:	0	π	$\frac{\pi}{2}$
x_0:	A	$-A$	0

$$x_0 = A\cos\phi$$

Also,

$$v_x = \frac{dx}{dt} = -\omega A\sin(\omega t + \phi)$$

$$a_x = \frac{dv_x}{dt} = \frac{d^2x}{dt^2} = -\omega^2 A\cos(\omega t + \phi)$$

$v_{max} = +\omega A$ and $v_{min} = -\omega A$

$a_{max} = +\omega^2 A$ and $a_{min} = -\omega^2 A$

$$v_{0x} = -\omega A\sin\phi$$

$$\frac{v_{0x}}{x_0} = \frac{-\omega A\sin\phi}{A\cos\phi} = -\omega\tan\phi \Rightarrow \phi = \tan^{-1}\left(\frac{-v_{0x}}{\omega x_0}\right)$$

$$A = \sqrt{x_0^2 + \frac{v_{0x}^2}{\omega^2}}$$

Reading for Class #38 4.25.24

14.3: Energy in SHM

ΣE in SHM is conserved

$E = K + U = \frac{1}{2} m v_x^2 + \frac{1}{2} k x^2 = C$

$\Rightarrow E = \frac{1}{2} m v_x^2 + \frac{1}{2} k x^2 = \frac{1}{2} k A^2 = C$

because at max A, $E = U$

Solved for v_x:

$\Rightarrow v_x = \pm \sqrt{\frac{k}{m}} \sqrt{A^2 - x^2}$

because object could be moving in either direction

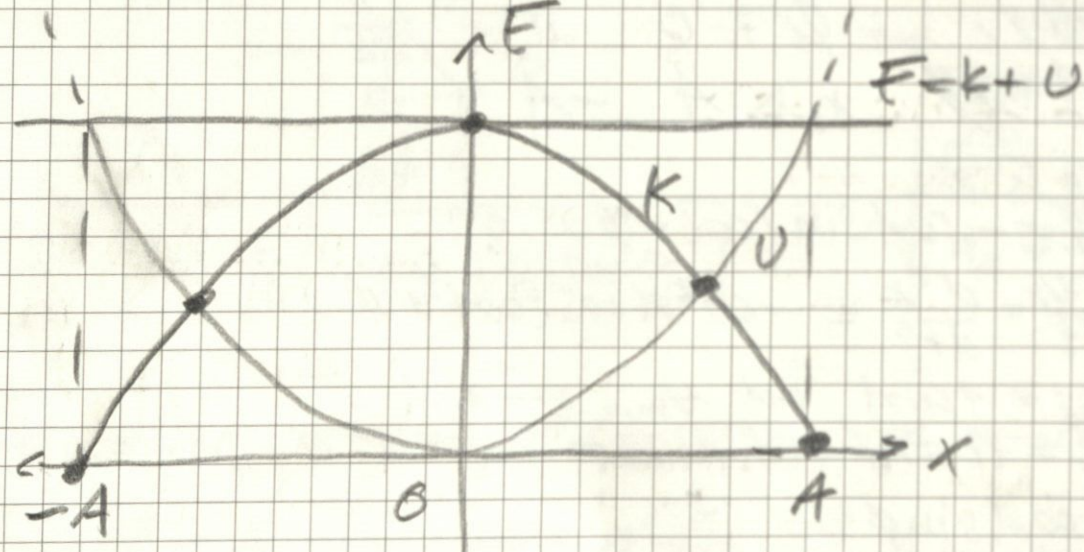
Maximum v_x: $x = 0$

$\Rightarrow v_{max} = \sqrt{\frac{k}{m}} A = \omega A$

because v_x oscillates b/w $-\omega A$ and ωA

Interpreting E, K, and U:

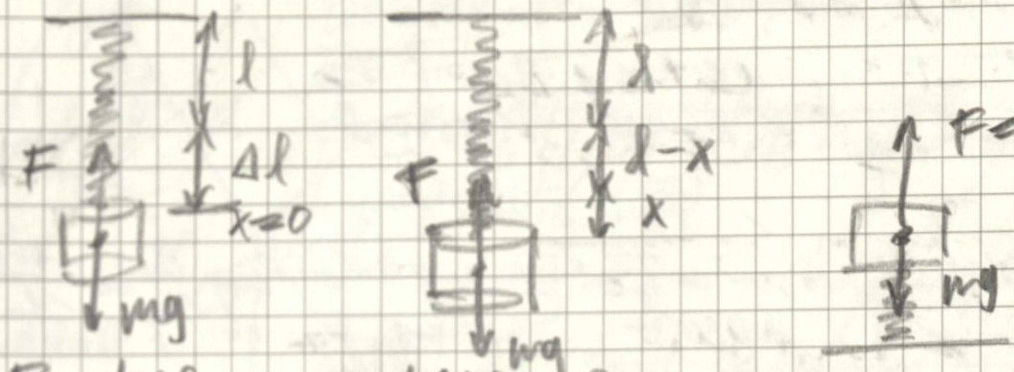
x	$-A$	$-\frac{1}{2}A$	0	$\frac{1}{2}A$	A
a_x	$a_x = a_{max}$	$a_x = \frac{1}{2} a_{max}$	$a_x = 0$	$a_x = -\frac{1}{2} a_{max}$	$a_x = -a_{max}$
v_x	$v_x = 0$	$v_x = \pm\sqrt{\frac{3}{4}}\, v_{max}$	$v_x = \pm v_{max}$	$v_x = \pm\sqrt{\frac{3}{4}}\, v_{max}$	$v_x = 0$
E	$E = U$	$E = K + U$	$E = K$	$E = K + U$	$E = U$



14.4: Applications of SHM

SHM models any $F_x = -kx$, not just ideal springs

Vertical SHM:



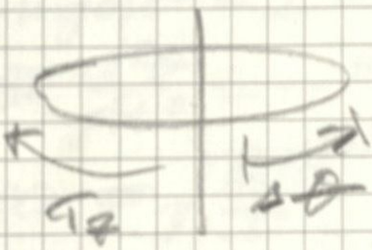
$F = k\Delta l$ $\quad$ $F = k(\Delta l - x)$

$\Rightarrow k\Delta l = mg \Rightarrow \sum F = k(\Delta l - x) + (-mg) = -kx$

Angular SHM:

$\tau_z = -\kappa\theta$

kappa, "torsion constant"



$\sum \tau_z = I\alpha_z = I\dfrac{d^2\theta}{dt^2}$

$\Rightarrow -\kappa\theta = I\alpha$

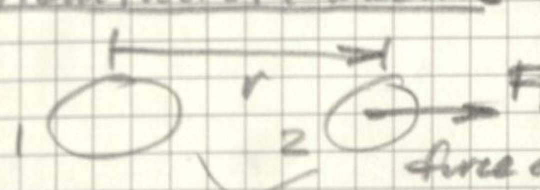
$\Leftrightarrow \dfrac{d^2\theta}{dt^2} = -\dfrac{\kappa}{I}\theta$

spring torque opposes angular displacement

$\omega = \sqrt{\dfrac{\kappa}{I}} \Leftrightarrow f = \dfrac{1}{2\pi}\sqrt{\dfrac{\kappa}{I}}$

$\theta = \Theta\cos(\omega t + \phi)$

Vibration of Molecules:



force exerted from 1 on 2

equilibrium: $r = R_0$

$U = U_0\left[\left(\dfrac{R_0}{r}\right)^{12} - 2\left(\dfrac{R_0}{r}\right)^{6}\right]$

positive constant SI: J

$r = R_0 \Leftrightarrow U = -U_0$

$r \to \infty \Leftrightarrow U = 0$

$F_r = -\dfrac{dU}{dr} = 12\dfrac{U_0}{R_0}\left[\left(\dfrac{R_0}{r}\right)^{13} - \left(\dfrac{R_0}{r}\right)^{7}\right]$

$x = r - R_0 \Leftrightarrow r = R_0 + x$, x: displacement from equilibrium

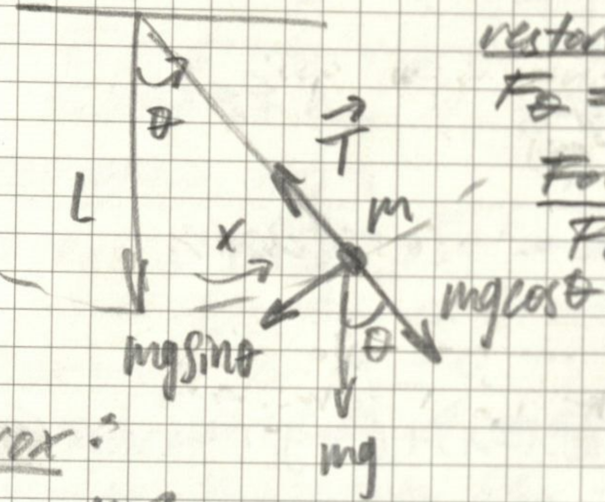
$\Rightarrow F_r \approx -\left(\dfrac{72U_0}{R_0^2}\right)x$

binomial approximation

Reading for Class #39: 4.28.24

14.5: The simple pendulum

idealised model of point mass suspended by massless, rigid string

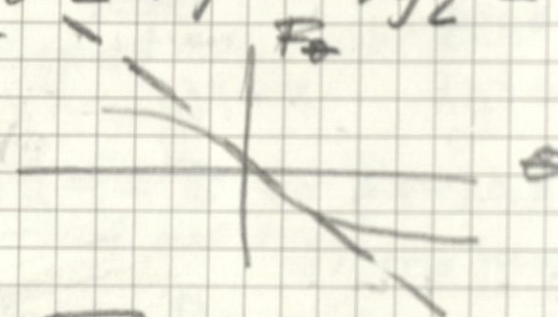


restoring force: F_θ

$F_\theta = -mg\sin\theta$

For small θ:

$F_\theta = -mg\theta = -mg\frac{x}{L} = -\frac{mg}{L}x$



Approx:

$k = \frac{mg}{L}$

$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{mg/L}{m}} = \sqrt{\frac{g}{L}}$

$f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{g}{L}}$

$T = \frac{2\pi}{\omega} = \frac{1}{f} = 2\pi\sqrt{\frac{L}{g}}$

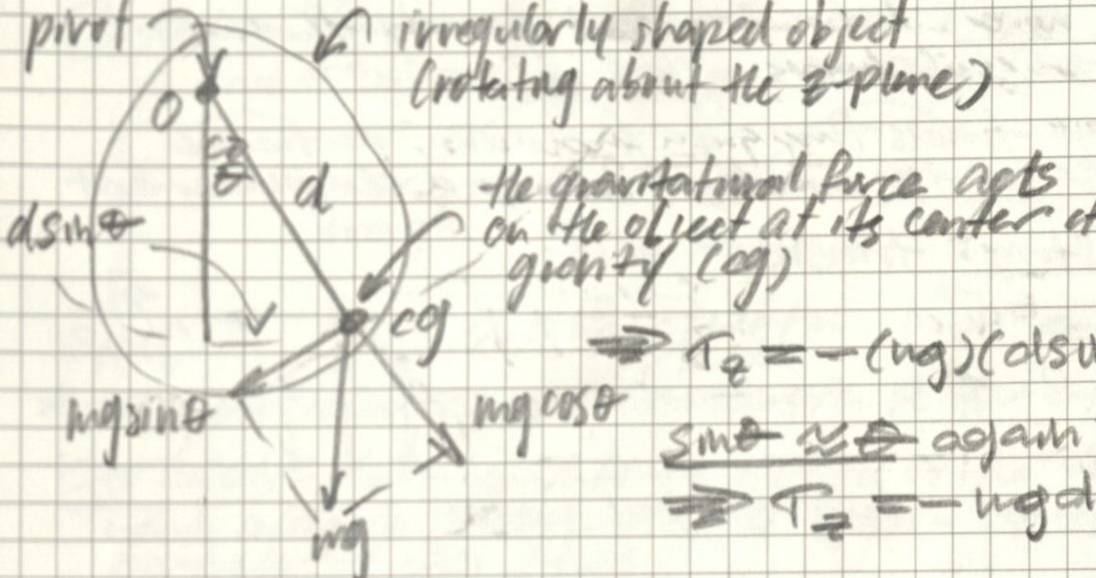
* all not dependent on m since F_θ proportional to m

Exact:

$T = 2\pi\sqrt{\frac{L}{g}}\left(1 + \frac{1^2}{2^2}\sin^2\frac{\theta}{2} + \frac{1^2 \cdot 3^2}{2^2 \cdot 4^2}\sin^4\frac{\theta}{2} + \dots\right)$

14.6: The Physical Pendulum

a real padelum that uses an extended object



$\Rightarrow \tau_z = -(mg)(d\sin\theta)$

$\underline{\sin\theta \approx \theta}$ again =

$\Rightarrow \tau_z = -mgd\theta$

$\sum \tau_z = I\alpha_z$

$-mgd\theta = I\alpha_z = I\frac{d^2\theta}{dt^2}$

$\Rightarrow \frac{d^2\theta}{dt^2} = -\frac{mgd}{I}\theta$

$\omega = \sqrt{\frac{mgd}{I}}$

$T = 2\pi\sqrt{\frac{I}{mgd}}$

d — distance from AoR to the <u>center of gravity</u> of the rod

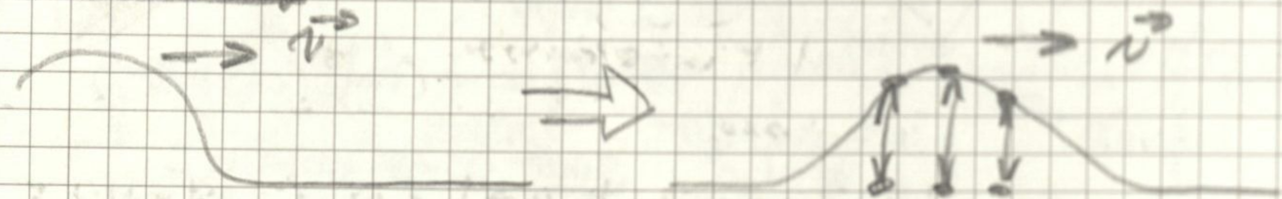
Reading for Class #10: 4.30.24

15.1: Types of Mechanical Waves

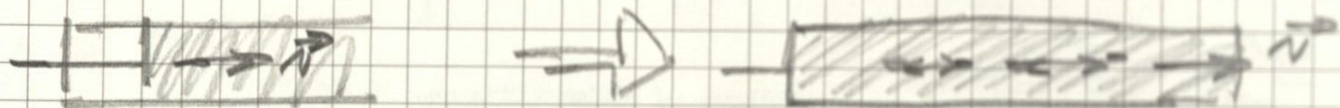
mechanical wave: disturbance that travels through a material/substance called a **medium**

- as a wave travels through a medium, particles that make up the medium undergo a displacement

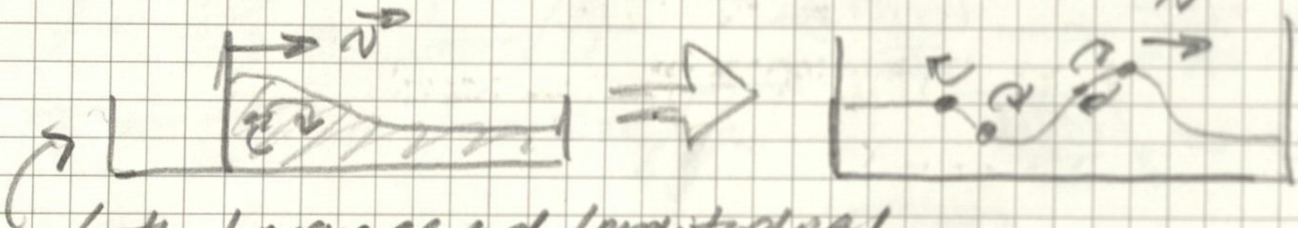
Wave on a String: transverse wave



Wave on a Fluid: longitudinal wave



Wave on the surface of a liquid:

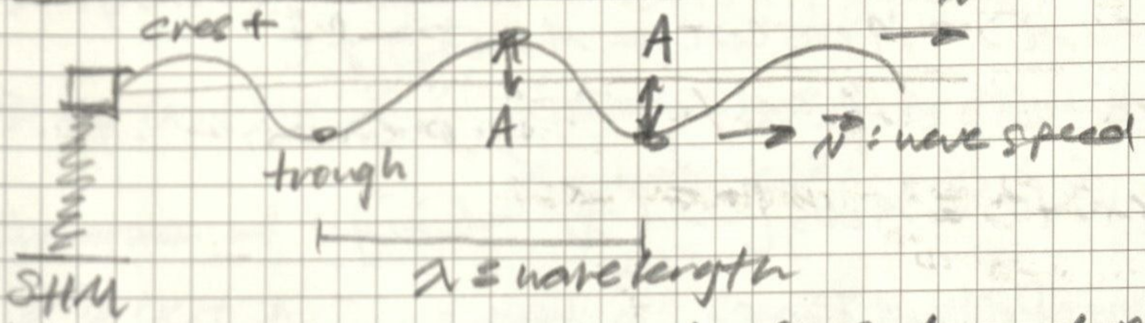


both transverse and longitudinal

15.2: Periodic Waves

Waves under periodic motion

Periodic Transverse Waves:

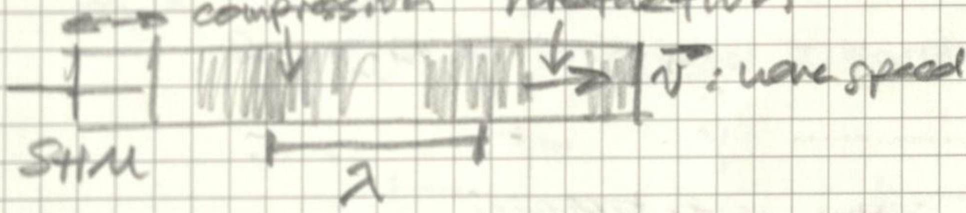


* The wave advances one wavelength λ for each period T

CAUTION: don't confuse the motion of the transverse wave along the string with the motion of the particles of the string

$v = \lambda f$

Periodic Longitudinal Waves:



also advances one λ every T

$v = \lambda f$

15.3: Mathematical Description of a Wave

Wave Function: $y(x,t)$

Sinusoidal Wave:

$$y(x=0,t) = A\cos\omega t = A\cos 2\pi f t$$

SHM with A, f, and $\omega = 2\pi f$

position

$$\Rightarrow y(x,t) = A\cos\left[\omega\left(\frac{x}{v} - t\right)\right]$$

wave speed

Has Many Different Forms:

$$\Rightarrow y(x,t) = A\cos\left[2\pi\left(\frac{x}{\lambda} - \frac{t}{T}\right)\right]$$

Wave Number: k

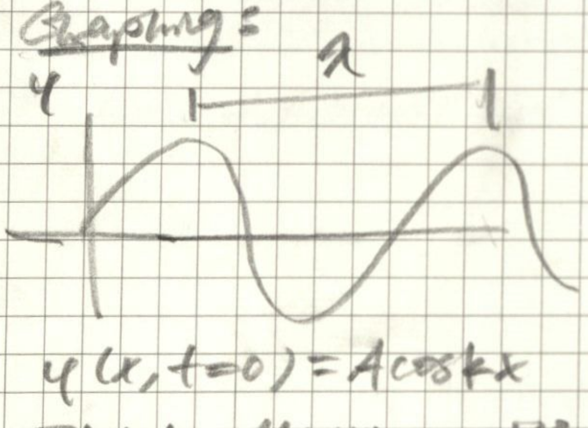
$$k = \frac{2\pi}{\lambda}$$

$$\omega = vk$$

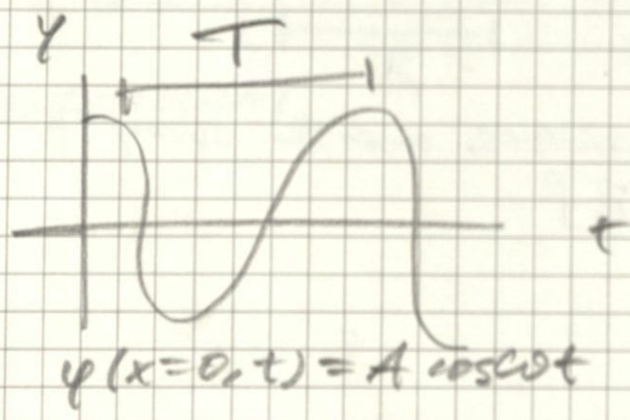
$$\Rightarrow y(x,t) = A\cos(kx - \omega t)$$

CAUTION: amplitude is independent of λ or f

Graphing:



$y(x,t=0) = A\cos kx$



$y(x=0,t) = A\cos\omega t$

If Wave Moving in $-x$:

$$y(x,t) = A\cos\left[\omega\left(\frac{x}{v} + t\right)\right] = A\cos\left[2\pi\left(\frac{x}{\lambda} + \frac{t}{T}\right)\right]$$

$$= A\cos(kx + \omega t)$$

Phase: $kx \pm \omega t = \phi(x,t)$

$$\frac{dx}{dt} = \frac{\omega}{k}$$

Velocity and Acceleration in Sinusoidal Waves:

$$y(x,t) = A\cos(kx - \omega t)$$

$$\Rightarrow v_y(x,t) = \frac{\partial y(x,t)}{\partial t} = \omega A \sin(kx - \omega t)$$

$$\Rightarrow a_y(x,t) = \frac{\partial^2 y(x,t)}{\partial t^2} = -\omega^2 A\cos(kx - \omega t)$$
$$= \omega^2 y(x,t)$$

Curvature of Wave:

$$\frac{\partial^2 y(x,t)}{\partial x^2} = -k^2 A\cos(kx - \omega t) = -k^2 y(x,t)$$

$$\Rightarrow \frac{\partial^2 y(x,t)/\partial t^2}{\partial^2_y(x,t)/\partial x^2} = \frac{\omega^2}{k^2} = v^2$$

Wave Equation:

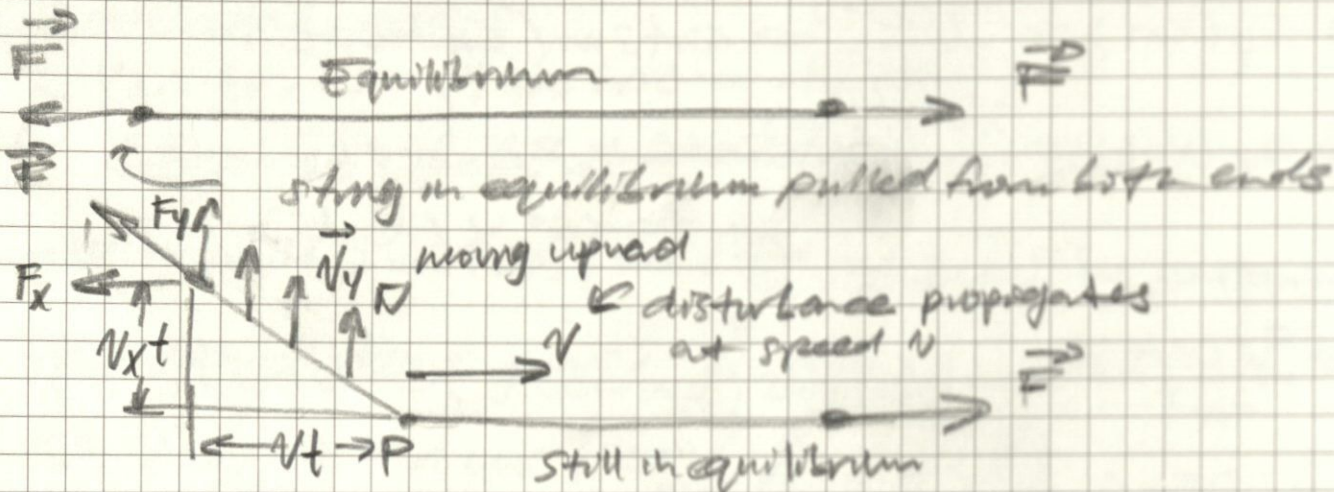
$$\frac{\partial^2_y(x,t)}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y(x,t)}{\partial t^2}$$

wave speed

Reading for Class #41:

5.3.24

15.4: Speed of a Transverse Wave



Transverse impulse = Transverse momentum

$\Rightarrow F_y t = m v_y$

Wave Speed:

$$\frac{F_y}{F} = \frac{v_y t}{v t} \Rightarrow F_y = F\frac{v_y}{v}$$

Transverse impulse = $F_y t = F\frac{v_y}{v} t$

Transverse momentum = $m v_y = (\mu v t) v_y$

(m of moving portion is the product of the mass per unit length μ and the length vt)

$$\Rightarrow F\frac{v_y}{v} t = \mu v t v_y \Rightarrow v = \sqrt{\frac{F}{\mu}}$$ ← tension in string ← mass per unit length

* can also be derived with Newton's Second Law and the Wave Equation

Speed of Any Mechanical Wave:

has same general form:

$$v = \sqrt{\frac{\text{Restoring force returning system to equilibrium}}{\text{Inertia resisting return to equilibrium}}}$$